Using phase diversity for the measurement of the complete spatiotemporal electric field of ultrashort laser pulses

Pamela Bowlan and Rick Trebino*

Georgia Institute of Technology, School of Physics, 837 State St. NW, Atlanta, Georgia 30332 USA
**Corresponding author: Rick.Trebino@physics.gatech.edu*

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Even so-called "complete" ultrashort laser pulse-measurement techniques actually have ambiguities and so are not truly complete. In particular, the spectral-interferometry technique called scanning SEA TADPOLE measures the "complete" spatiotemporal intensity and phase of arbitrary ultrashort pulses (using a previously characterized spatially uniform reference pulse), but the difficulty of maintaining the stability of the required interferometer to submicron resolution while scanning in space usually blurs the frequency-independent spatial component of the pulse phase. We show here, however, that this information is actually still contained in the measured SEA TADPOLE data, and using a simple Gerchberg–Saxton-like phase-diversity algorithm, it can be recovered from measurements in only two planes, yielding a truly complete spatiotemporal measurement of the pulse field, limited only by any possible ambiguities present in the reference pulse. © 2012 Optical Society of America

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1. INTRODUCTION

The measurement of light is one of the most important endeavors in science, and much progress has occurred recently in the measurement of light with ultrafast variations [1]. The ultimate goal of optical measurement technology is the measurement of the complete spatiotemporal field of light, $E(x,y,z,t)$. Such measurement technology is important in view of currently evolving techniques for the spatiotemporal shaping of ultrashort pulses [2–5], which have potential applications to biomedical imaging, micromachining, and a wide range of other fields. In addition, efforts to attain ultrahigh intensity for such important applications as laser nuclear fusion will be stymied if undesired variations in the pulse intensity or phase versus space or time are present, limiting the actual intensity at the target. Unfortunately, measuring separate pulse and beam shapes is insufficient, as common optical elements, such as prisms and lenses, introduce spatiotemporal distortions, in which the temporal field depends on position. These effects necessitate the measurement of the complete spatiotemporal field, $E(x,y,z,t)$, often with submicron spatial resolution and fs temporal resolution. To solve this problem, we recently introduced a measurement technique called scanning SEA TADPOLE, which can, in principle, measure the complete four-dimensional electric field, $E(x,y,z,t)$ of ultrashort pulses with the required spatial and temporal resolutions [6–11].

SEA TADPOLE is a linear-optical spectral-interferometric technique that involves spatially sampling the unknown field with an optical fiber or a near-field scanning microscopy (NSOM) fiber probe, yielding a spatial resolution approximately equal to the mode size of the fiber, which can be as small as ~50 nm. This sample of the unknown field is interfered with a previously characterized, spatially uniform reference pulse in a spectrometer, so that $E(\omega)$ can be determined for the fiber location (x,y,z). Then, in order to measure the spatial dependence of the field, the fiber is scanned in the transverse and longitudinal (propagation) directions, so that $E(x,y,z,\omega)$ is measured at each fiber position (x,y,z), and, as a result, the complete spatiotemporal field $E(x,y,z,\omega)$ is obtained. This quantity can be Fourier transformed to yield the complete field versus space and time $E(x,y,z,t)$. SEA TADPOLE is also convenient and easy to use because it lacks the precise alignment required for most interferometric methods, and, as it does not involve placing the fiber tip in close proximity to a surface, it lacks the fiber-damage problems of NSOM. We have used SEA TADPOLE to measure tightly focused pulses, superluminal Bessel pulses, and pulses diffracted by various apertures, all with ultrahigh spatiotemporal resolution [6–11].

As we have observed previously, however, SEA TADPOLE scans in space and uses a fiber interferometer, so it is afflicted with a slow phase drift (~1 rad/s) due to the inherent difficulty of maintaining optics to the required submicron accuracies while scanning them and also due to small refractive-index changes in the fibers [6]. Thus, a measurement of $E(x,y,\omega)$, which can require a minute or more, randomizes the spatial phase, that is, the zeroth-order spectral-phase coefficient versus transverse position, $\varphi_0(x,y,\omega_0)$, in the expansion of the spectral phase versus frequency:

$$\varphi(x,y,\omega) = \varphi_0(x,y,\omega_0) + (\omega-\omega_0)\varphi_1(x,y,\omega_0) + \frac{1}{2}(\omega-\omega_0)^2\varphi_2(x,y,\omega_0) + \ldots \quad (1)$$

Figure 1 shows a measurement of the observed drift versus time for the first three terms in the above Taylor series for the spectral phase at a given point in space, illustrating that only the zeroth-order, non-frequency-dependent term is

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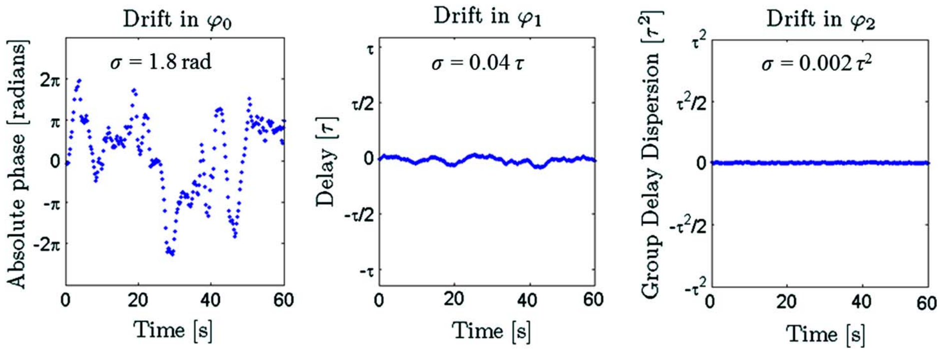

Fig. 1. (Color online) Measurement of the drift in the (a) absolute phase, φ_0, (b) delay, φ_1, and (c) group delay dispersion, φ_2, in a typical SEA TADPOLE measurement. The rms variation of each of the curves, σ, is given with respect to the pulse duration τ. While the phase drift in our interferometer randomizes the absolute phase, φ_0, all higher-order, frequency-dependent terms are unaffected and accurately measured.

significantly randomized. All higher-order terms are accurately and stably measured.

This drift did not affect the results of our previous studies, in which $\varphi_0(x, y, \omega_0)$ was not of interest, and only intensities and the frequency-dependent component of the spatiotemporal phase were required.

In these studies and in SEA TADPOLE measurements in general, one simply measures the spatiotemporal field at each z plane of interest (we typically chose nine values of z). However, if the spatial phase is also known, the field $E(x, y, \omega)$ can be numerically propagated from one plane to another and, in principle, one can avoid having to directly measure the spatiotemporal field for all values of z, vastly simplifying the measurement. So it would be very helpful to be able to measure or retrieve it.

This problem is related to a number of retrieval problems in purely spatial optics involving monochromatic beams, in which one or more intensities versus position are measured, but no information about the spatial phase is measured at all [12]. If the spatial intensity is measured at only one plane, the problem is called the phase-retrieval problem, and it can be solved (the spatial phase, $\varphi_0(x, y, \omega_0)$, can be retrieved) if the problem is two-dimensional and even a fairly weak constraint is available, such as that the intensity versus x and y is zero outside a finite area in a Fourier plane [12]. If the spatial intensity is measured at two planes related by a Fourier transform (that is, Fraunhofer diffraction), the problem is even better behaved, and the well-known Gerchberg–Saxton (GS) algorithm robustly retrieves the phase, independent of the dimensionality of the problem [13]. It involves simply Fourier transforming back and forth between the two domains, replacing the intensity versus position by the measured quantity in each domain until convergence occurs. Finally, if the spatial intensity is measured at two planes not related by a Fourier transform, but instead by the more general Fresnel transform (sometimes referred to as a fractional Fourier transform), the problem is called "phase diversity," and algorithms analogous to the GS algorithm, but using the Fresnel integral, can be used to find the phase [14].

In monochromatic-light problems, once the spatial intensity and phase are known in one plane (value of z), the diffraction integral can find them both everywhere (for all values of z). On the other hand, for pulses, rather than monochromatic beams, the problem is more complex because the spatial phase must be known for all frequencies present in the inherently nonmonochromatic pulse. And the relative spectral phase $[\varphi_0(x_0, y_0, \omega)]$ of each monochromatic component of the pulse at some point (x_0, y_0) must also be specified. We have previously introduced and demonstrated a simple holographic technique (which we call STRIPED FISH) that performs just such measurements—and does so on a single shot [15–17]. The spatial resolution of STRIPED FISH, however, is limited to the pixel size of the camera used, so it cannot measure pulses with submicron spatial resolution, as can SEA TADPOLE. So it is worth determining how to measure the complete spatiotemporal field in SEA TADPOLE.

What kind of retrieval problem does SEA TADPOLE correspond to? SEA TADPOLE measures the "complete" intensity and phase versus frequency at numerous spatial coordinates, but, as previously mentioned, the zeroth-order phase in the expansion versus frequency, $\varphi_0(x, y, \omega_0)$, is lost due to mechanical instability and drifts. This corresponds to the spatial phase for each frequency present in the pulse, but these phases are necessarily the same for all frequencies, since the zeroth-order phase in an expansion with respect to frequency, by definition, is independent of frequency. But this quantity can vary with position, x and y, so this function of x and y must then be retrieved.

How can the spatial phase be retrieved in SEA TADPOLE? It is a phase-diversity problem, but a multicolor one, in which we know not only the intensity versus the position at two or more planes, but also all the other phase coefficients at those planes as well. Thus it is a particularly easy one, most of the work having been done in the measurements.

Here we show that we can solve the SEA TADPOLE phase-diversity problem and recover the spatial phase using standard phase-diversity ideas, making even an unstable SEA TADPOLE measurement a true complete measurement of the spatiotemporal field of an arbitrary pulse. Specifically, to reconstruct the complete spatiotemporal field, we need only measure the spatiotemporal (or, equivalently, the spatiospectral) intensity and phase (minus, of course, the spatial phase) at one plane and the spatiospectral intensity at another. No phase information is required at all at the second (or any other) plane.

We should point out that other techniques have been introduced for measuring the spatiotemporal intensity and phase of pulses that do not randomize the spatial phase [18–20]. Our work is not the first use of the GS algorithm for measuring the spatiotemporal field of ultrashort laser pulses. Faccio and coworkers used it in an elegant approach they call "CROAK," which involves two intensity measurements in Fourier planes in conjunction with a frequency-resolved-optical-gating (FROG) measurement for additional relative-phase information [21, 22]. In one implementation [21], they measured the intensity versus the transverse position (x) and the frequency and also versus the transverse angle (θ or k_x) and the frequency and performed a one-dimensional GS phase retrieval. In another, they measured the intensity versus x and t and also versus k_x and ω and performed two-dimensional GS phase retrieval [22]. None of these methods, however, achieves submicron spatial resolution. Also, in our experience, redundancy in the data, which occurs in some of the above methods, but is especially the case in scanning SEA TADPOLE, is very helpful in exposing and eliminating systematic error, which is very common in measurements of such ephemeral events as ultrashort light pulses.

2. METHOD

Figure 2 illustrates our algorithm. For our initial guess for the field, we use the measured pulse field at the first plane z_1 with its correct intensity and higher-order phase, but whose zeroth-order (spatial) phase consists of random values. We propagate this field (for each wavelength, λ, or frequency, ω) to the next plane, z_2. There we replace the spatiospectral amplitude with the measured amplitude and then back-propagate the field to the plane z_1, where we again replace the spatiospectral amplitude with the measured one. This is repeated until the rms difference between the measured and propagated amplitudes is minimal, which indicates that the current spatial phase is the correct value. Unlike the traditional GS algorithm [23], we (1) use the nonparaxial version of the angular spectrum of plane waves (ASPW) and (2) propagate the fields from one plane to another arbitrary plane [24,25] (rather than the Fourier transform plane). This allows for measurements from any two planes to be used (not just the plane just before the lens and its focal plane, as in traditional GS geometries). We

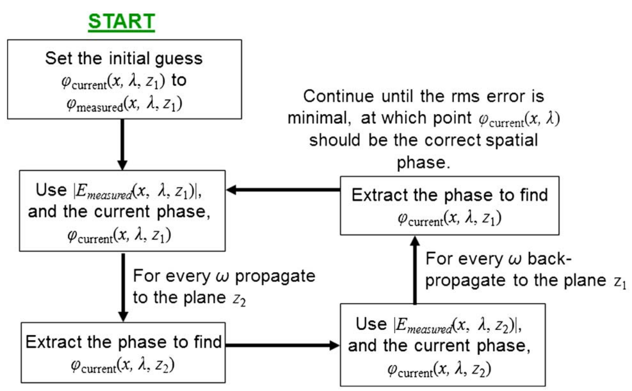

Fig. 2. (Color online) SEA TADPOLE spatial-phase-retrieval algorithm. We use the measured spatiospectral intensity and phase, $E(x, \lambda, z_1)$, at one plane, z_1, and the spatiospectral amplitude at another, z_2, to recover the spatial phase $\varphi(x, y, \omega_0)$. On the first iteration, we use the measured spatiotemporal amplitude and phase for the field at z_1, whose spatial phase is random. This field is propagated to the plane z_2, where we replace the spatiospectral amplitude with the measured amplitude and use the spatiospectral phase of the propagated field. Then we back-propagate this to plane z_1, replacing the spatiospectral amplitude with the measured amplitude at z_1. This is repeated until the rms difference between the amplitudes of the propagated and measured fields is minimal.

used the nonparaxial ASPW approach because it works well for numerical apertures as high as 0.7. Of course, for weak focusing, the nonparaxial approach is not necessary.

We assume only one transverse spatial coordinate for simplicity and because our previous measurements involved beams with cylindrical symmetry. We typically measured $E(x, \lambda)$ at $y = 0$ (at nine or more different values of z, the distance from the focus). But this approach should also be valid for a measurement versus both transverse spatial coordinates. Because of the cylindrical symmetry in our problem, the ASPW reduces to a Hankel transform, which we perform using the algorithm of Guizar-Sicairos and Gutiérrez-Vega [26] using the code they kindly provided. Also, unlike standard GS algorithm applications, in which only intensities are known, we measure the spatiospectral intensity and the phase versus wavelength to all orders except the zeroth in multiple planes. Thus, we have much more information, making our problem much easier, although we chose not to use all of this additional information in finding the unknown spatial phase and instead have used it here to confirm that we have in fact found the correct spatial phase.

3. EXPERIMENTAL RESULTS

To test our algorithm, we used our previous measurements of a focusing pulse from a 0.09 NA plano-convex lens [6]. In the algorithm, we used the fields $E(x, \lambda)$ measured at $z_1 = -1.1$ mm and $z_2 = +0.7$ mm, where the positions are given relative to the position of the focus ($z = 0$) and whose measured amplitudes are shown in Fig. 3. The images in the second column in Fig. 3 show the intensities obtained by propagating the field from plane z_1 to z_2 and vice versa using the measured spatial phase (i.e., one iteration). The disagreement between these and the measured intensities illustrates the problems with the (incorrect) measured spatial phase.

After 29 iterations, however, our algorithm converged with an rms difference between the measured and propagated amplitudes of 1.48%. The images in the third column of Fig. 3 are the propagated amplitudes using the retrieved spatial phase and are in agreement with the measured images, illustrating a successful recovery of the spatial phase.

The rms difference between the measured and propagated amplitude at $z = 0.7$ mm is shown at the right of Fig. 3, illustrating the quick convergence of the algorithm.

To further test our results, we numerically propagated the field using the corrected spatial phase from $z = -1.1$ mm to seven other planes. These results, shown in Fig. 4, are in good agreement with the measured spatiospectra, shown above them.

We do not show the retrieved spatial phase because it was slowly varying and uninteresting. The initial guess for the spatial phase was quite complex (and also uninteresting, as it consisted of random values). We can conclude that the phase-diversity algorithm is evidently powerful in view of the large difference between these functions.

Our results illustrate that, by using a simple GS-like phase-diversity algorithm, we can determine the complete electric field of ultrashort pulses from our scanning SEA TADPOLE measurements. Note that we could use the measured spatiospectral amplitudes at more than one plane in the algorithm, but we found that this was not necessary, and the rms error obtained by using the measured pulses at nine planes was no

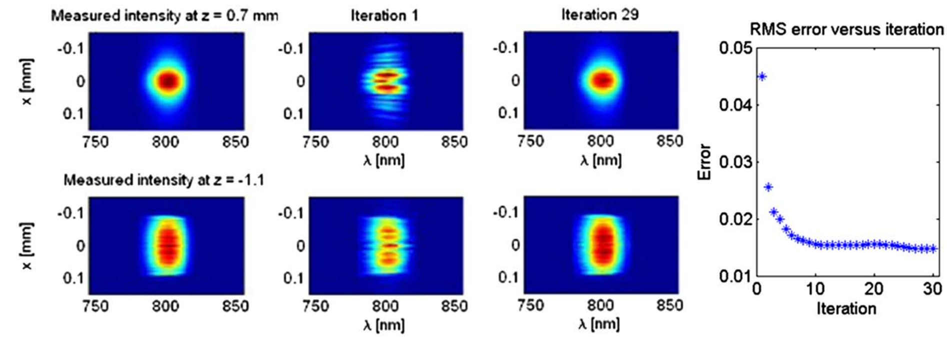

Fig. 3. (Color online) Spatial phase retrieval results. First column: measured spatiospectral amplitudes at z_1 (top) and z_2 (bottom). Second column: spatiospectral intensities obtained by propagating the field at z_2 to z_1 (top) and vice versa (bottom) using the measured spatial phase after only one iteration. Third column: same as the previous, but using the retrieved spatial phase after 29 iterations. Note the excellent agreement between the first and last columns.

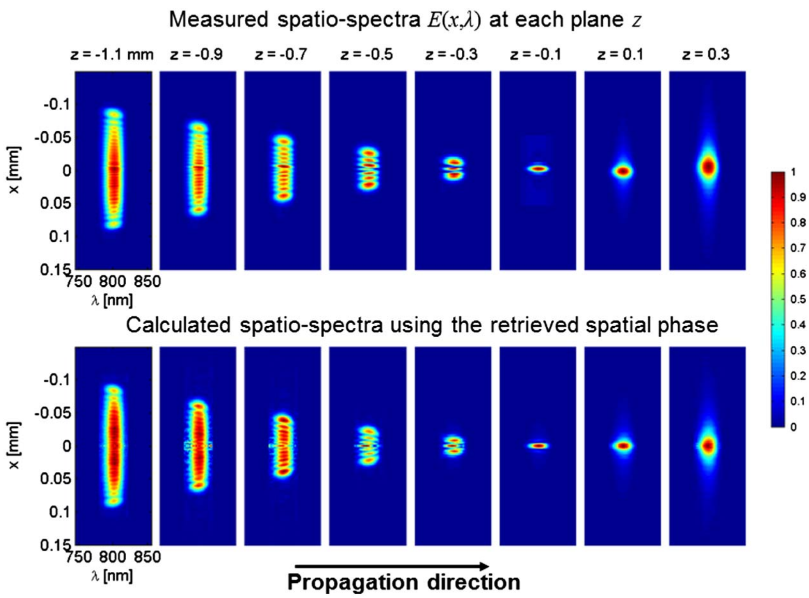

Fig. 4. (Color online) Measured (top) and numerically propagated spatiospectral intensities using the field from $z = -1.1$ mm, including the retrieved spatial phase.

lower than that obtained using only two. We did find that the algorithm converged the fastest when we used one plane before the focus and another after the focus (as we have done above).

4. DISCUSSION AND CONCLUSIONS

SEA TADPOLE is a linear interferometric method for measuring the spectral phase of an unknown pulse using a spatially uniform, previously measured reference pulse. By scanning a fiber across the unknown beam, the spectral phase can be measured at every position, yielding the spatiotemporal field, $E(x, y, \omega)$ [or equivalently $E(x, y, t)$]. But without actively stabilizing the interferometer, which uses fibers, the measured spatial phase is blurred due to a slow drift. Here we have shown that the spatial-phase information is actually present in the data, as long as the spatiotemporal field is measured at at least two longitudinal positions (values of z). We demonstrated that, using a GS-like phase-retrieval algorithm, the spatial phase can be recovered by numerically propagating the spatiotemporal field back and forth between the two planes, and replacing the spatiospectral intensity with the measured quantity. But unlike the GS algorithm, rather than using the Fourier transform, we use a (nonparaxial) version of the diffraction integral to propagate the fields, so that any two planes can be used. Using measurements of a ~$0.1 \times$ NA spherical lens, we recovered the spatial phase in less than 30 iterations of the algorithm. To illustrate that we correctly recovered the spatial phase, we propagated the corrected field $E(x, y, t)$ to seven other planes, where we had also measured the spatiotemporal field, and the propagated intensity was in good agreement with the measured intensity.

This simple extension of SEA TADPOLE allows for the true complete spatiotemporal field $E(x, y, z, t)$ to be determined by measuring the field without the spatial phase (i.e., without the zeroth-order spectral phase versus the transverse position) at one plane and the spatiospectral intensity at another plane.

This approach could also be used in conjunction with a wide range of other pulse-measurement techniques that only measure the field versus time. For example, making individual FROG measurements [1] of every small spatial region of a beam in two planes with the help of an aperture smaller than the beam—or, better, a small nonlinear-optical particle

[14]—yields a similar spatial-phase retrieval problem. However, if the pulse-measurement method uses the pulse to measure itself, rather than using a previously characterized reference pulse as in SEA TADPOLE, the unknown pulse arrival time versus transverse position, which is the first-order spectral-phase coefficient, $\varphi_1(x, y)$, also goes unmeasured and so would need to be retrieved as well.

Of course, all such measurements could be limited by ambiguities in the measurement of the reference pulse, such as possible nonmeasurement of its absolute (zeroth-order) phase. In any case, with the possible exception of STRIPED FISH, the combination of SEA TADPOLE and the phase-diversity approach that we have described herein is, to our knowledge, the most complete spatiotemporal pulse-measurement technique ever developed. It certainly provides the best spatial resolution by more than an order of magnitude or more.

Finally, it must be admitted that, even if the reference pulse were completely ambiguity-free, no measurement technique is truly complete in the most general sense of the term. While SEA TADPOLE can measure an arbitrary field versus space and time, we have implicitly assumed a scalar field, that is, a polarized beam, and our computations (but not our device) are currently limited to numerical apertures of 0.7. Of course, two separate measurements at different (orthogonal) polarizations could suffice to obtain the complete polarization dependence, and more general propagation code could achieve even higher numerical apertures. However, at such tight focusing, the on-axis field is thought to have a longitudinal component, which SEA TADPOLE currently does not appear to be able to measure, providing at least one interesting challenge for future work.

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