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Spatial chirp in ultrafast optics

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Abstract

Spatial chirp is a common spatio-temporal effect, but its parameterization is currently unsatisfactorily vague. In this paper, we propose and compare two definitions of spatial chirp, which we call ''spatial dispersion'' and ''frequency gradient''. The appropriate definition to use depends on the application. For Gaussian beams and pulses, the relationship between the two definitions is found to be analogous to that between the definitions of temporal chirp in the time and frequency domains.

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A beam with ''spatial chirp'' has its different frequency components separated in space transverse to the propagation direction. It is a very common and often undesirable spatio-temporal distortion in ultrafast optics and can be introduced by many routine operations in laser laboratories. For example, a beam passing through an element with angular dispersion, such as a prism or a grating, experiences angular dispersion; after additional propagation, the frequency components naturally separate in space, resulting in spatial chirp. A second prism or grating, aligned anti-parallel to generate negative group-delay dispersion [1], removes the angular dispersion, but significant spatial chirp remains (Fig. 1(a)). Although using the prism/grating pair in a double-pass arrangement can eliminate spatial chirp in the output beam, small and almost inevitable misalignments often allow some residual spatial chirp to remain in the beam. Other common practices in a laboratory, such as propagating a beam through a tilted substrate (Fig. 1(b)), also introduce spatial chirp.

On other occasions, researchers deliberately separate different frequency components spatially, such as in Fourier-synthesis pulse shaping [2,3]. In this technique, a lens (or curved mirror) is placed

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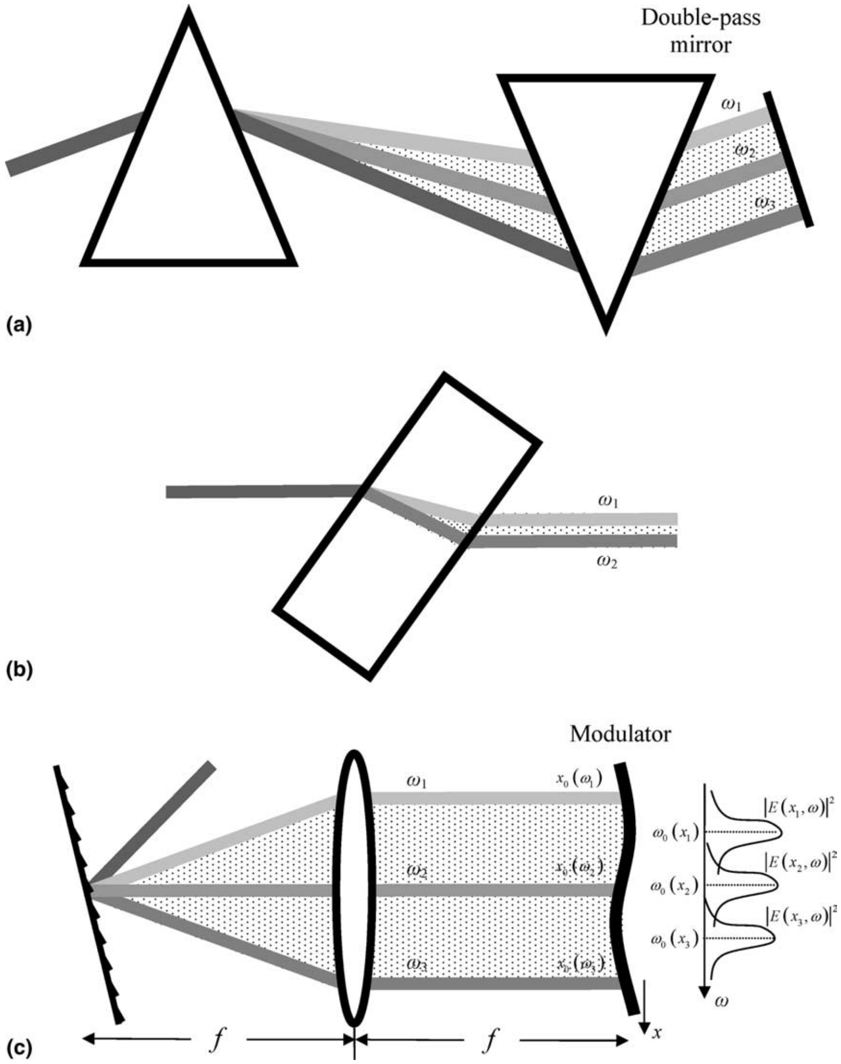

Fig. 1. Generation of spatial chirp in a: (a) prism pair, (b) tilted substrate and (c) *f–f* Fourier-synthesis pulse shaper.

one focal length away from a grating (or prism) in a telecentric configuration, mapping frequency to position, that is, introducing spatial chirp, at its focal plane (Fig. 1(c)). The accuracy of pulse shaping depends on the degree of spatial chirp at the focal plane, on which extensive studies have been carried out [2,4,5]. Other applications of spatial chirp include the suppression of longitudinal mode competition [6,7] in the laser design.

The 21st century has seen intensified interest in the spatio-temporal analysis of ultrashort-pulse beams. Numerous methods have been proposed to measure spatio-temporal characteristics of an ultrafast laser beam [8–15]. To study these spatio-temporal effects, clear and unambiguous definitions of the various coupling parameters are undoubtedly required. In the past, discussions of spatial chirp, one of the most common spatio-temporal coupling effects, have been mostly confined to specific devices [4,5,7,16–20], and its definition has been somewhat arbitrary and vague in the literature. In this paper, we attempt to clarify the meaning of this term. Specifically, we show that there are two different definitions of spatial chirp, which we refer to as "spatial dispersion" and "frequency gradient." Which definition to use depends on the situation. For Gaussian beams and pulses, we find the relationship between these two parameters to be analogous to that between the parameters describing temporal chirp in the time and frequency domains.

We begin with the case where no spatial chirp is present, and the amplitude of the electric field at position x and frequency ω (defined as frequency offset from the center frequency of the beam) can be written in the form:

$$E(x, \omega) = E_x(x)E_\omega(\omega),$$

where the spatial amplitude $E_x(x)$ and the spectral amplitude $E_\omega(\omega)$ are completely separate and potentially arbitrary functions of one variable. [1]

In the presence of spatial chirp (here we assume that it exists in one transverse spatial dimension x only), $E(x, \omega)$ becomes an inseparable two-variable function, where the spatial and spectral dependences are coupled. We can easily measure the spatio-spectral intensity profile of the spatially chirped beam by sending the beam into an imaging spectrometer with a two-dimensional camera on its output image plane, as depicted in Fig. 2. Fields sampled at different points along the entrance slit of the spectrometer are spectrally resolved onto different rows of the camera image, resulting in a trace of intensity in the x–ω domain. With linear spatial chirp, the spatio-spectral intensity profile will appear tilted. Fig. 3(a) shows a typical x–ω intensity plot of an experimental beam with spatial chirp.

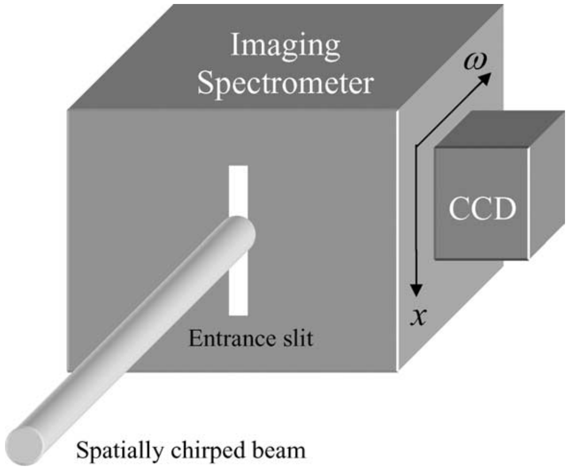

Fig. 2. Measuring spatial chirp using an imaging spectrometer.

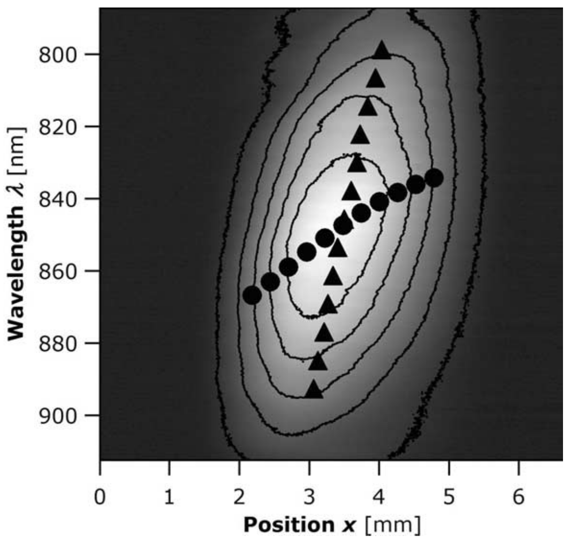

Fig. 3. Measured spatio-temporal intensity profile of an experimental spatially chirped beam. The line of triangles indicates the $\omega_0(x)$ function, which determines frequency gradient, and the line of circles indicates the $x_0(\omega)$ function, which determines spatial dispersion.

Obviously, the degree of spatial chirp can be characterized by measuring the tilt of the x–ω trace. However, there is a subtlety in this measurement, namely, that there are two intuitive,

[1] An equivalent representation of a general spatio-temporal ultrashort-pulse beam is the space–time Wigner function [21], a four-dimensional real-valued distribution function, which carries the same information about the ultrashort-pulse beam as the complex spatio-temporal (or spectral) field expression. The various two-dimensional marginals of the space–time Wigner function are the expressions of pulse-beam intensity in these domains. Although the space–time Wigner function is a powerful tool in the study of ultrafast beams in space and time, we choose not to use it in our analysis, because this work only involves studying the beam intensity in space and frequency. For that purpose, the simpler spatio-spectral field expression is a more appropriate tool.

but different, ways of measuring the tilt of the trace in the x–ω plane. The first involves measuring the center frequency, ω_0, of each spatial slice, which yields a function $\omega_0(x)$. The slope of the $\omega_0(x)$ function, $\upsilon \equiv \mathrm{d}\omega_0/\mathrm{d}x$, is a natural measure of spatial chirp, which we will call the *frequency gradient*. The other means of parameterization involves measuring the beam center position, x_0, of each frequency component, which yields the function, $x_0(\omega)$. Its slope $\zeta \equiv \mathrm{d}x_0/\mathrm{d}\omega$ is also a valid measure of spatial chirp, which we will call the *spatial dispersion*. Both parameters characterize the spatial chirp, and very importantly, they are not trivial reciprocals of one another. In the absence of spatial chirp, both parameters are zero. And as can be seen in Fig. 3, the lines of triangles and circles do not overlap.

A few researchers have been aware of this subtlety of spatial chirp parameterization. Ohmae et al. [20] noted the difference between the $\omega_0(x)$ and $x_0(\omega)$ curves in their analysis of a Martinez-type multi-pass pulse stretcher, and their particular ray-tracing calculation yields the $x_0(\omega)$ result. However, there has been no previous work published on the general relationship between the two spatial chirp parameters, which is necessary background for the increasingly important research now occurring in many laboratories on spatio-temporal distortions. We will devote the rest of the paper to this issue and will draw an analogy between spatial chirp and temporal chirp at the end, which we believe will shed new light on their physical implications.

First, we would like to point out that in most cases spatial chirp is introduced through angular dispersion; therefore, spatial dispersion is often the more fundamental of the two definitions. When a beam with angular dispersion $\beta = \mathrm{d}\theta_0/\mathrm{d}\omega$ propagates a distance L, the induced change in spatial dispersion is

$$\Delta\zeta = L\beta,$$

which is completely determined by the optical system only. Frequency gradient, on the other hand, is affected indirectly. As can be seen later, the change of the frequency gradient depends not only on the optical system, but on the parameters of the input beam and pulse as well. It is in this sense that spatial dispersion is a more fundamental parameter of spatial chirp in its generation, manipulation, and removal, although frequency gradient is often more useful in the intended application of spatial chirp. In short, both quantities are important.

The relationship between frequency gradient and spatial dispersion is in general complicated in that it depends on the spatial-mode profiles of all the constituent frequency components, and the shape of spectrum. A common assumption is to assign all the frequency components the same spatial-mode profile, which we will write as $E_x(x)$. We will also write the complex spectral amplitude of the beam as $E_\omega(\omega)$. Then the field expression at position x and frequency ω in the beam can be written in terms of spatial dispersion ζ as:

$$E(x,\omega) = E_\omega(\omega)E_x(x-\zeta\omega). \tag{1}$$

We will focus on the simplest possible case, which is a Gaussian spectrum and a Gaussian spatial profile for all the frequency components. Namely,

$$\begin{aligned} E_\omega(\omega) &= \exp\left[-\left(\frac{\omega}{\Delta\omega}\right)^2\right], \\ E_x(x) &= \exp\left[-\left(\frac{x}{\Delta x}\right)^2\right], \end{aligned} \tag{2}$$

where $\Delta\omega$ is the frequency bandwidth of the beam (1/e amplitude half width); Δx is the beam width of a particular frequency component.

The spatio-spectral field amplitude for a pulse with spatial dispersion is then

$$E(x,\omega) = E_0 \exp\left[-\left(\frac{\omega}{\Delta\omega}\right)^2\right] \exp\left[-\left(\frac{x-\zeta\omega}{\Delta x}\right)^2\right]. \tag{3}$$

We may reorganize the two exponential functions and write the field in terms of frequency gradient υ. The expression becomes

$$E(x,\omega) = E_0 \exp\left[-\frac{x}{(\Delta x')^2}\right] \exp\left[-\left(\frac{\omega-\upsilon x}{\Delta\omega'}\right)^2\right] \tag{4}$$

where

$$\upsilon = \frac{\zeta}{\zeta^2 + \left(\frac{\Delta x}{\Delta\omega}\right)^2} \quad \text{is the frequency gradient;} \tag{4a}$$

$$\Delta\omega' = \left[\frac{1}{(\Delta\omega)^2} + \frac{\zeta^2}{(\Delta x)^2}\right]^{-(1/2)}$$

is the locally reduced frequency bandwidth due to spatial chirp, available at any particular locations in the beam; (4b)

$$\Delta x' = \left[\frac{1}{(\Delta x)^2} - \left(\frac{\upsilon}{\Delta\omega}\right)^2\right]^{-(1/2)}$$

is the increased overall beam width due to spatial chirp. (4c)

Eq. (4a) describes the relationship between the frequency gradient υ and the spatial dispersion ζ. Note that they are not reciprocals of each other. In fact, they are asymptotically reciprocals only when spatial dispersion $\zeta \equiv dx_0/d\omega$ is much larger than $\Delta x/\Delta\omega$. If spatial dispersion ζ is very small, on the other extreme, the two parameters are actually proportional. For a given beam width Δx and frequency bandwidth $\Delta\omega$, frequency gradient υ reaches its maximum achievable value $\frac{1}{2}\left(\frac{\Delta\omega}{\Delta x}\right)$ when $\zeta \equiv \frac{dx_0}{d\omega} = \frac{\Delta x}{\Delta\omega}$. Fig. 4 shows the relationship of frequency gradient and spatial chirp with $\Delta x = 1.0$ mm and $\Delta\omega = 0.094$ rad/fs, the conditions for the experimental trace in Fig. 3.

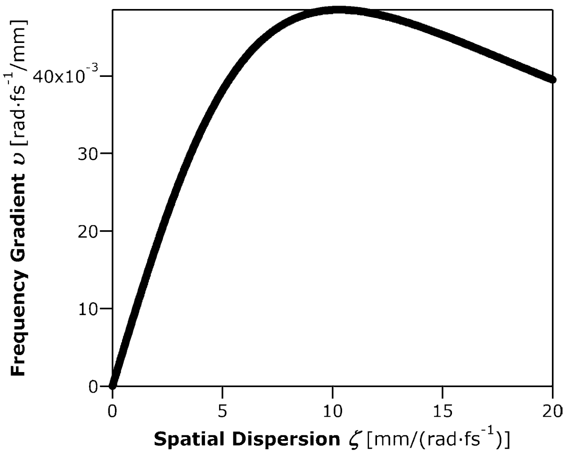

Fig. 4. Theoretical plot of frequency gradient vs. spatial dispersion, with the same experimental conditions for the measurement in Fig. 3.

The distinction between the two definitions of spatial chirp is quite analogous to that between the definitions of temporal chirp in time and frequency domains. We can describe a linearly chirped Gaussian pulse either in the time domain,

$$E(t) = |E(t)|\exp[-i\phi(t)] = E_0 \exp\left[-\left(\frac{t}{\Delta t}\right)^2\right]\exp\left(-\frac{i}{2}\phi_2 t^2\right) \quad (5)$$

or equivalently in the frequency domain,

$$\tilde{E}(\omega) = \left|\tilde{E}(\omega)\right|\exp[-i\varphi(\omega)] = \tilde{E}_0 \exp\left[-\left(\frac{\omega}{\Delta\omega}\right)^2\right]\exp\left(-\frac{i}{2}\varphi_2\omega^2\right). \quad (6)$$

The two expressions are a Fourier transform pair.

The physical significance of temporal chirp parameters ϕ_2 and φ_2 can be viewed as such: In the time domain, $-\phi_2$ is the derivative of instantaneous (angular) frequency $\omega_0 \equiv -(d\phi(t)/dt) = -\phi_2 t$ with respect to t. On the other hand, in the frequency domain, φ_2 (often called group-delay dispersion) is the derivative of group delay $t_0 \equiv (d\varphi(\omega)/d\omega_0) = \varphi_2\omega$ with respect to ω. Parameters $\phi_2 = -d\omega_0/dt$ and $\varphi_2 = dt_0/d\omega$ are two different, but equivalent, parameters describing temporal (spectral) chirp in the time/frequency domains, just as parameters $\zeta = dx_0/d\omega$ and $\upsilon = d\omega_0/dx$ are the parameters describing spatio-temporal chirp in the frequency/space domains. Indeed, the relationship between ϕ_2 and φ_2 is:

$$\varphi_2 = \frac{-\phi_2}{\frac{1}{4}\phi_2^2 + \frac{1}{(\Delta t)^2}}, \quad (7)$$

which follows from the Fourier transform and is remarkably similar to the relationship between ζ and υ (Eq. (4b)) for the case of spatial chirp.

The experimental process that introduces temporal chirp determines whether ϕ_2 or φ_2 is the more fundamental parameter for a given situation. For example, propagation through a linear dispersive material will add φ_2 phase term to the electric field in frequency domain. The field in the time domain, found by inverse Fourier transform, will then show pulses that are temporally longer (or shorter). On the other hand, self-phase modulation adds a ϕ_2 phase term in the time domain. Fourier transforming to the frequency domain will then yield a broader spectrum.

Likewise, the various occurrences of spatial chirp require a consideration of one spatial-chirp parameter or the other. For example, in pulse shaping, frequency gradient determines the mapping of spatial modulation to spectral modulation. In addition, previously, we showed that single-shot frequency-resolved-optical-gating (FROG) devices for measuring ultrashort laser pulse intensity and phase, including the very simple device GRENOUILLE, also measure frequency gradient [14]. However, other optical devices, including pulse stretchers and compressors, are best modeled using spatial dispersion. From our definition, we can see that the two parameters are related in a complicated way, involving both the beam width and the frequency bandwidth. Indeed, there is a maximum frequency-gradient value one can achieve with given pulse and beam parameters. Knowing the relationship between these two parameters should help achieve better control of experimental conditions involving spatial chirp.

To conclude, we have proposed and compared two definitions of spatial chirp, namely, spatial dispersion and frequency gradient. We derived the relationship between the two parameters, and we find it analogous to that between the two quadratic-phase parameters (ϕ_2 and φ_2) characterizing temporal chirp in the time/frequency domains.

References

[1] O.E. Martinez, J.P. Gordon, R.L. Fork, J. Opt. Soc. Am. A 1 (1984) 1003.

[2] J.P. Heritage, A.M. Weiner, R.N. Thurston, Opt. Lett. 10 (1985) 609.

[3] A.M. Weiner, J.P. Heritage, E.M. Kirschner, J. Opt. Soc. Am. B 5 (1988) 1563.

[4] M.M. Wefers, K.A. Nelson, IEEE J. Quantum Electron. 32 (1996) 161.

[5] M.M. Wefers, K.A. Nelson, J. Opt. Soc. Am. B 12 (1995) 1343.

[6] I.P. Christov, N.I. Michailov, M.B. Danailov, Appl. Phys. B 53 (1991) 115.

[7] N.I. Michailov, J. Opt. Soc. Am. B 9 (1992) 1369.

[8] Z. Sacks, G. Mourou, R. Danielius, Opt. Lett. 26 (2001) 462.

[9] C. Dorrer, E.M. Kosik, I.A. Walmsley, Appl. Phys. B 74 (2002) S209.

[10] C. Dorrer, I.A. Walmsley, Opt. Lett. 27 (2002).

[11] L. Gallmann, G. Steinmeyer, D.H. Sutter, T. Rupp, C. Iaconis, I.A. Walmsley, U. Keller, Opt. Lett. 26 (2001) 96.

[12] T. Tanabe, H. Tanabe, Y. Teramura, F. Kannari, J. Opt. Soc. Am. B 19 (2002) 2795.

[13] S. Akturk, M. Kimmel, P. O'Shea, R. Trebino, Opt. Express 11 (2003) 491.

[14] S. Akturk, M. Kimmel, P. O'Shea, R. Trebino, Opt. Express 11 (2003) 68.

[15] M. Raghuramaiah, A.K. Sharma, P.A. Naik, P.D. Gupta, Opt. Commun. 223 (2003) 163.

[16] O.E. Martinez, J. Opt. Soc. Am. B 3 (1986) 929.

[17] O.E. Martinez, Opt. Commun. 59 (1986) 229.

[18] R. Thurston, J. Heritage, A. Weiner, W. Tomlinson, IEEE J. Quantum Electron. 22 (1986) 682.

[19] A. Efimov, C. Schaffer, D.H. Reitze, J. Opt. Soc. Am. B 12 (1995) 1968.

[20] G. Ohmae, T. Yagi, K. Nanri, T. Fujioka, Jpn. J. Appl. Phys. 39 (2000) 5864.

[21] J. Paye, A. Migus, J. Opt. Soc. Am. B 12 (1995) 1480.