Nontrivial Ambiguities in FROG? Fortunately, Not.

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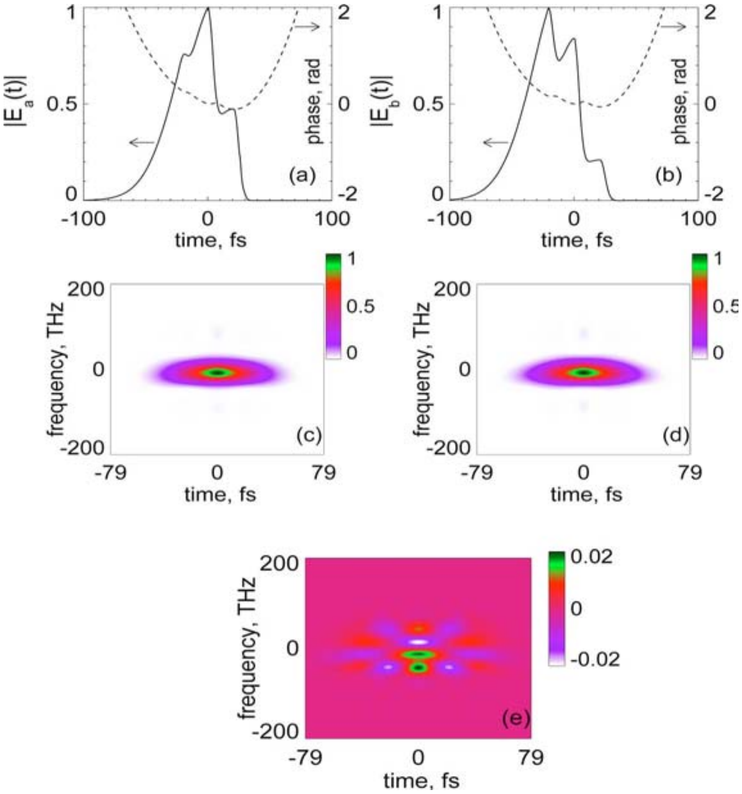
A December 2007 Optics Letter[1] reported two nontrivial "ambiguities" in second-harmonic-generation (SHG) frequency-resolved-optical-gating (FROG). And a December 2008 "Erratum" on this paper by the same authors[2] reiterated this claim and the conclusions of the initial publication (it reported no errors). However, the first "ambiguity" is clearly wrong—the result of computational error by the authors of that paper (errors repeated in the "erratum"). The other is well-known, trivial, and common to most pulse-measurement techniques (except for XFROG and SEA TADPOLE). It is also easily removed in FROG (but not in other methods) using a simple, well-known FROG variation. Finally, these authors' main conclusion—that autocorrelation can be more sensitive to pulse variations than FROG—is also wrong. The following article is an expanded version, including figures, of a one-page Comment that has been accepted for publication in Optics Letters, and which will appear soon. It is reprinted here with the permission of the editor.

The most important characteristic of any measurement technique is the avoidance of ambiguities. Alas, all ultrashort-pulse measurement techniques have ambiguities. Fortunately, all known ambiguities in FROG are *trivial* (unimportant or easily removed). In their December 2007 Optics Letter, however, Yellampalle, Kim, and Taylor (YKT)[1] claim to have found a *nontrivial* ambiguity in SHG FROG: two pulses with different substructure, whose SHG FROG traces they claim cannot be distinguished in practice. Computing the traces' rms difference (usually called the FROG error), they report a tiny value: $G = 7 \times 10^{-6}$, indicative of an ambiguity.



YKT Fig. 3. a. Pulse #1. b. Pulse #2. c, d. SHG FROG traces of pulses #1 and 2. e. the difference between the two traces. Note that the difference is about ~ 2% over 10% of the trace and hence clearly about .002, not .000007, as reported by YKT.

Unfortunately, this value is wrong. In fact, $G = 2.4 \times 10^{-3}$. A quick glance at YKT's plot (YKT Fig. 3e, shown at right) of the trace difference, which is ~ 2% over 10% of the trace area and near zero elsewhere, easily confirms this value. It is likely that YKT neglected to take the square root in computing the rms difference.

In an early version of our Comment (evidently leaked to these authors by one of the reviewers), we considered the possibility that their error was due to their having used an unrealistically large array and so having included numerous meaningless additional zeros in their average. Also, we pointed out that perhaps they could have avoided such an error by using an alternative version of the rms error—one normalized by the nonzero trace area. This is the error over only the nonzero region of the trace, and it is called G'. In their "erratum" (evidently a Reply to this preliminary version of our Comment, even though it hadn't been published yet!), YKT then computed G' and obtained 8×10^{-4}, unfortunately, again wrong. The correct value is $G' = 2.6\%$. Again, this larger value is consistent with their Fig. 3, in which the difference between the traces is about 2% over the nonzero area of the trace. Again it appears that they neglected to take the square root in computing the rms.

Whichever error is computed, such traces are easily distinguished in practice.

More importantly, simply quoting a difference between two FROG traces is naive. That, of course, is all that can be done in autocorrelation-based methods. FROG, on the other hand, enjoys a powerful pulse-retrieval algorithm. Thus the issue is *not* how the traces appear to the eye, or even their difference, but whether the *pulses retrieved from them* would be confused.

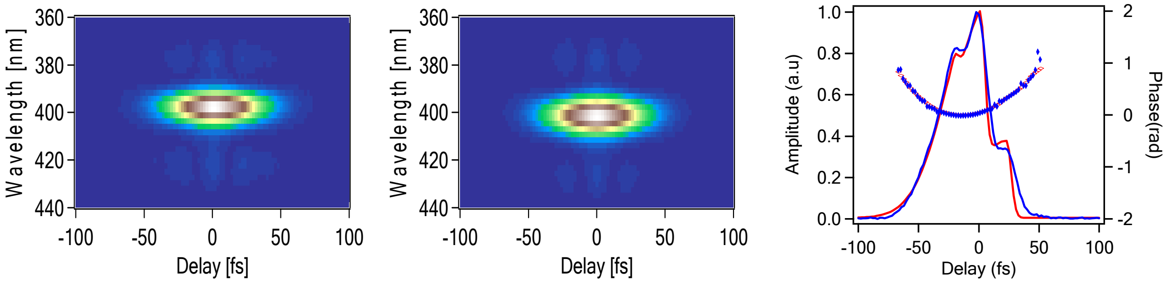


Fig.1. From left: SHG FROG trace of YKT's pulse #1 with 1% noise added, retrieved SHG FROG trace, and the generated and retrieved pulses in the time domain. The red curve indicates the generated pulse and the blue curve indicates the retrieved pulse. The initial guess for the algorithm was the "ambiguous" pulse (pulse #2). The array size was 128 x 128, the FROG G error of the retrieval is 0.0036, and the (intensity-weighted) G' error is 0.0824.

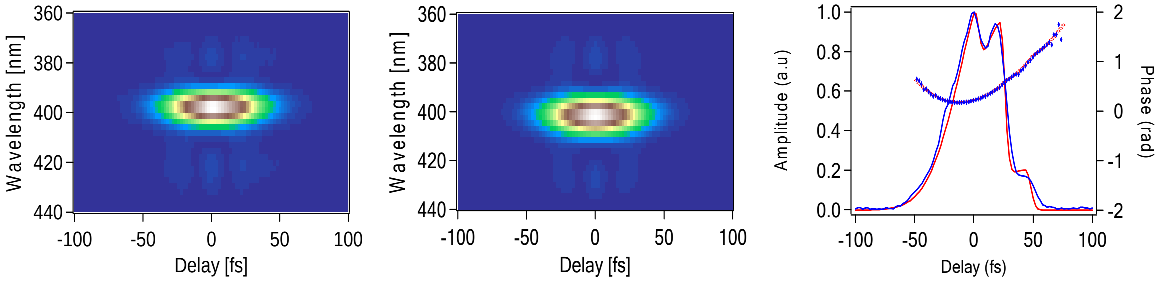


Fig.2. From left: SHG FROG trace of pulse #2 with 1% noise added, retrieved SHG FROG trace and the generated and retrieved pulse in the time domain. The initial guess for the algorithm was the "ambiguous" pulse (pulse #1). The FROG G error of the retrieval is 0.004, and the G' error is 0.0803.

To test whether the two pulses are ambiguities, we generated SHG FROG traces of the two "ambiguous" pulses and added up to 2% additive noise to simulate a very noisy experiment. We ran the usual SHG FROG algorithm using random noise as the initial guess. Also, to attempt to fool the algorithm, we also used the *other "ambiguous" pulse* as the initial guess in each case. Despite this attempt at deception, the algorithm achieved excellent and rapid convergence to the correct pulse in all cases. Clearly, such pulses are not ambiguities in SHG FROG (or any other version of FROG).

YKT also reminded us of a trivial SHG FROG ambiguity, described earlier, by one of the authors herself[3, 4] and also by one of us.[5, 6] It involves pulses well-separated in time (YKT Fig. 1). It's well known that relative phases, amplitudes, and directions of time (DOT) for well- separated pulses or modes confuse most pulse-measurement techniques.[5-7] But SEA TADPOLE and a FROG variation, XFROG, easily avoid them.[7] Also, in our paper on the issue[5, 6] (and strangely not mentioned by YKT), we also showed how to *remove* all such ambiguities and also SHG FROG's DOT ambiguity: using an *etalon* for the beam splitter yields an easily measured train of overlapping pulses. Such a train of pulses is easily measured by FROG, and retrieving the individual waveform (E) from the train (E_{train}) is trivial:

$$E(t) = E_{train}(t) - \varepsilon E_{train}(t-T),$$

where T is the round-trip time of the etalon and ε is the ratio of field strengths of successive individual pulses in the train. This method also removes the overall DOT ambiguity in SHG FROG and in addition automatically calibrates any FROG device. We called it Procedure for Objectively Learning the Kalibration And Direction Of Time (POLKADOT) FROG. In Figs. 3 and 4, we show how this approach easily removes the ambiguity in the case of the double pulses mentioned by YKT.

In general, intensity and interferometric autocorrelation are not appealing alternatives to FROG. It is well known that pulses (including all those of YKT) *cannot* be retrieved from either type of autocorrelation trace, even when additional measures (such as the spectrum) are included, unless arbitrary assumptions are made or the pulse is trivially simple.[5, 8] The complexity of the mathematics in autocorrelation in autocorrelation prevents even knowing the ambiguities. Finally, both types of autocorrelation traces blur features as pulses become more complex, clearly losing much information and so rendering them *fundamentally* unable to measure complex pulses. FROG traces, on the other hand, grow appropriately more complex, thus retaining the necessary information about the pulse (see Figs. 5 and 6). Indeed, FROG easily measures and retrieves extremely complex pulses without ambiguity.[5, 9] This cannot be said of any other technique available, except for XFROG and SEA TADPOLE, which also work very well, but which require reference pulses.

Acknowledgments

We thank Reviewer #1 for actually obtaining the pulses from the authors (who refused to provide them to us) and also for independently confirming our calculations.

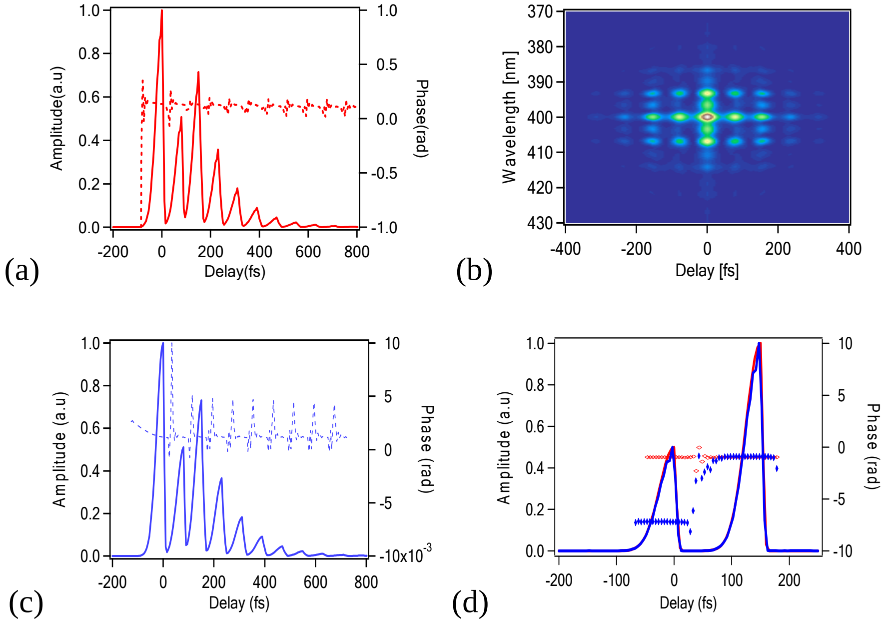


Fig. 3. (a) the double pulse train after the etalon, (b) the SHG FROG trace of the etalon-transmitted pulse train, (c) the retrieved pulse train from the trace, (d) the original generated double pulse and the double pulse retrieved using $E(t) = E_{train}(t) - \varepsilon E_{train}(t\text{–}T)$. The solid line indicates the generated pulse and the dashed line indicates the retrieved pulse. The FROG G error of the retrieval is 0.00027, and the G' error is 0.0056.

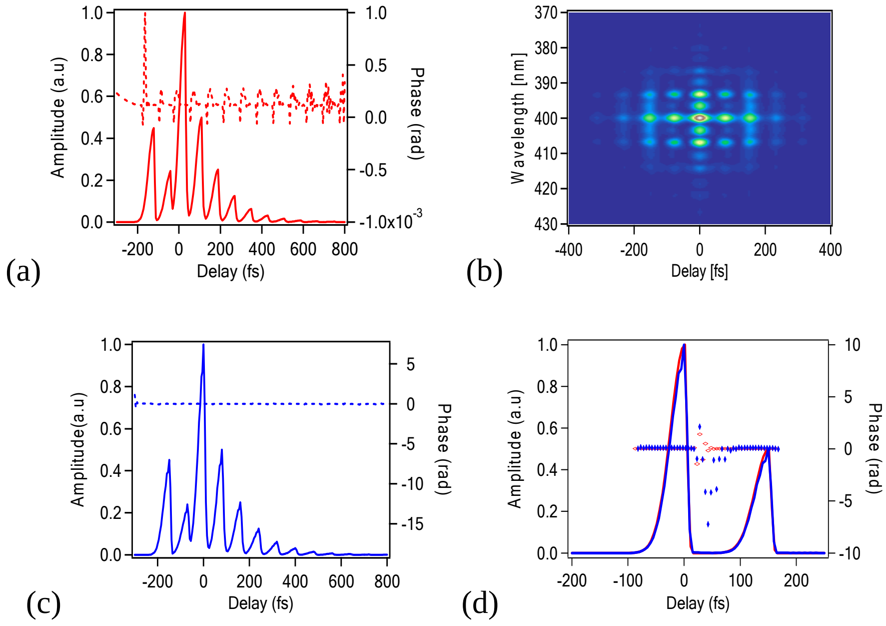


Fig. 4. (a) the double pulse train after the etalon, (b) the SHG FROG trace of the etalon-transmitted pulse train, (c) the retrieved pulse train from the trace, (d) the original generated double pulse and the double pulse retrieved using $E(t) = E_{train}(t) - \varepsilon E_{train}(t\text{–}T)$. The solid line indicates the generated pulse and the dashed line indicates the retrieved pulse. The FROG G error of the retrieval is 0.00024, and the G' error is 0.0049.

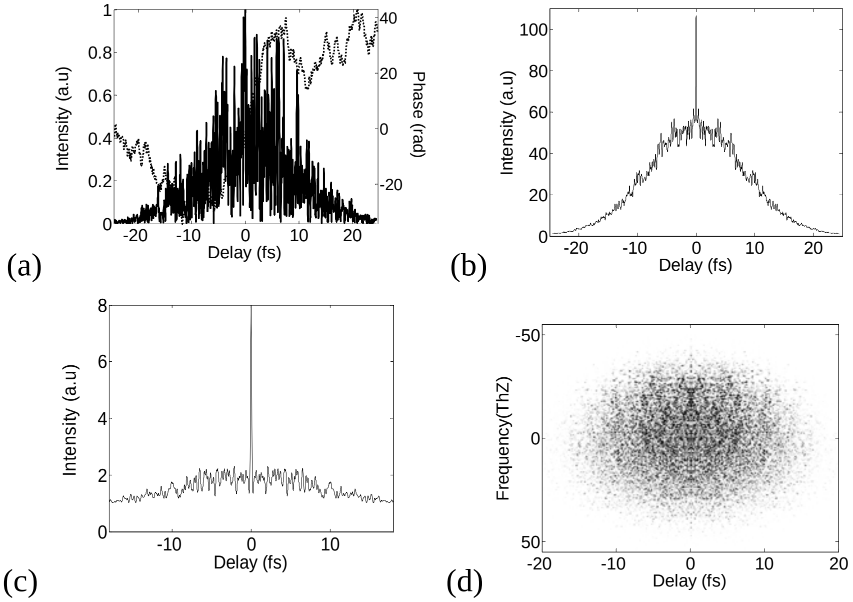


Fig. 5. (a) Generated complex pulse with TBP of 475, (b) Intensity autocorrelation trace of this complex pulse, (c) Interferometric autocorrelation trace of this complex pulse, (d) SHG FROG trace of this complex pulse. While the structure (which contains the pulse information) in the autocorrelation and interferometric autocorrelation is nearly washed out, the highly complex structure in the FROG trace has a visibility of close to 100%.

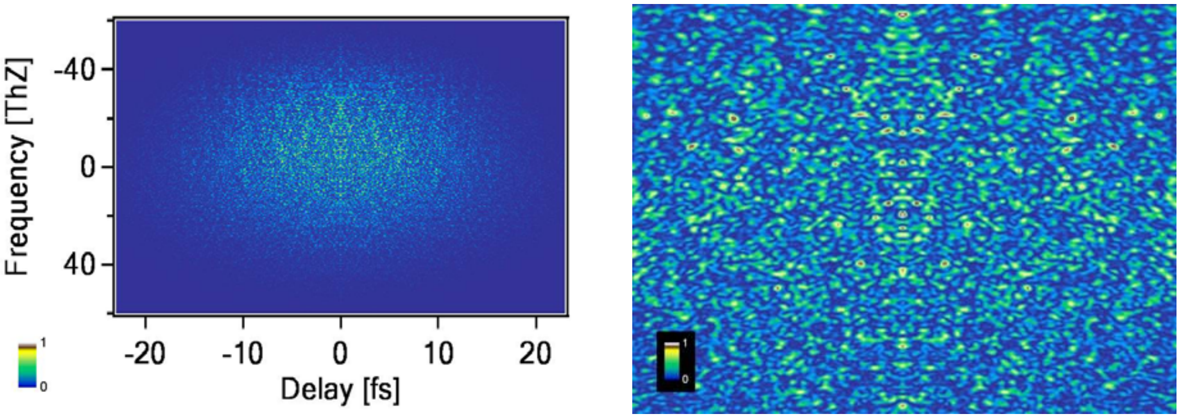


Fig. 6. Left: False-color SHG FROG trace of the complex pulse of Fig. 5. Right: expanded view of the non-zero region of the trace. Note the highly complex structure in the FROG trace, which has a structure visibility of close to 100%.

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