

complex pulses, MUD TADPOLE's typically simple reference pulse only overlaps in time with a fraction of the temporal length of the unknown pulse and makes spatial fringes only with that temporal piece of the unknown pulse. Fourier-transforming the resulting trace with respect to position (as in SEA TADPOLE) and keeping only the ac term at the spatial-fringe frequency yields the pulse intensity and phase of the temporal piece of the unknown pulse that temporally overlaps with the reference pulse.

Varying the delay of the reference pulse yields SEA TADPOLE traces for all temporal pieces of the long unknown pulse and so yields the complete intensity and phase of every temporal piece of the pulse. Concatenating in time all these measured pieces of the pulse reconstructs the entire pulse in time for as long a pulse as one can generate delays for.

As an aside, in MUD TADPOLE, it is important to remember that the reference pulse lengthens significantly in time inside the spectrometer, specifically, to the spectrometer's inverse spectral resolution. So for each delay, it actually measures a fairly long temporal piece of the unknown pulse in one measurement. For example, a 20fs reference pulse measures a temporal piece of the unknown pulse ~10ps long when using a readily available spectrometer with ~100GHz spectral resolution. Since a MUD TADPOLE measurement uses multiple reference pulses to oversample information at each time value, the delay spacing of successive reference pulses would only need to be ~3ps, rather than 20fs.

The effective spectral resolution of MUD TADPOLE is therefore many times that of SEA TADPOLE and many times the spectral resolution of the spectrometer. Specifically, it is the reciprocal of the reference-pulse *delay range*. In other words, it can measure pulses as long as the delay that can be generated. Since it is much easier to generate large delays than to improve spectral resolution, this is a significant advantage, akin to that of Fourier-transform spectrometers over grating spectrometers, but without the stringent alignment issues of such devices and with the ability to also measure the spectral phase of the unknown pulse.

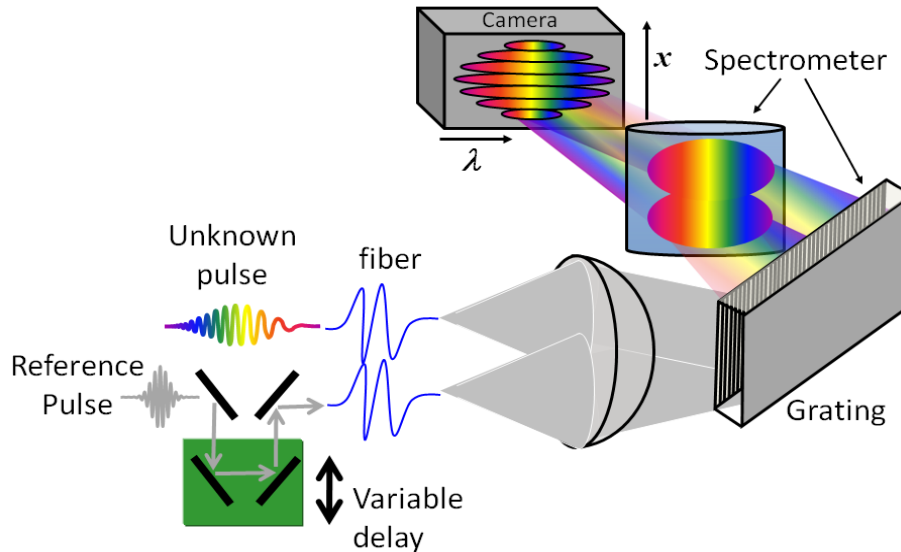


Fig. 1. Experimental setup for MUD TADPOLE. Both the unknown pulse and the reference pulse are coupled into two equal-length single-mode fibers. The reference pulse passes through a delay stage, which provides the variable delay. In the horizontal dimension, the light is collimated by the spherical lens and spectrally resolved by the spectrometer. In the vertical dimension, the beams cross at a slight angle, resulting in spatial fringes at the camera.

The most complex pulses measured so far using MUD TADPOLE have had TBPs of ~700 [17]. So here we extend MUD TADPOLE to even more complicated pulses and consider its limits with regard to complex-pulse measurement.

2. Time-bandwidth product limits

The maximum TBP that an ordinary spectrometer can measure is its finesse (its spectral range divided by its resolution). This is equal to the amount of information in the measured spectrum. In the limit of noise-free measurements, the maximum TBP that MUD TADPOLE can measure is the same quantity, but with the spectral resolution replaced by the inverse delay range, which would be MUD TADPOLE's finesse.

MUD TADPOLE's maximal TBP is, however, limited by another experimental factor: the dynamic range of the camera. This is because, as the reference pulse only makes spatial fringes with the temporal piece of the unknown pulse with which it temporally overlaps, the rest of the unknown pulse also inevitably impinges on the camera, yielding a spatially structureless background of no value to that particular measurement and which must therefore be filtered out numerically. While the relevant Fourier filtering works very well, this background could become very large for very complex pulses, which require many many reference pulses. Thus, the dynamic range of the camera used in the spectrometer poses a limit to the largest TBP measurable by MUD TADPOLE. Fortunately, all that is necessary is to measure the phase of the spatial fringes against this background, and this can be done quite sensitively, even with as little as an oscillation amplitude of a fraction of a count, against a large constant background. Using one count as the limit, we may estimate that the largest TBP measurable by a MUD TADPOLE apparatus is the product of the finesse of the spectrometer and the dynamic range of the camera used to make the measurement. If the camera is chosen to match the spectrometer, that is, its number of columns is equal to the spectrometer finesse, then the maximal TBP measurable with MUD TADPOLE is the product of the number of columns (or rows, whichever is greater) and its dynamic range.

The best commercially available cameras, to our knowledge, have a dynamic range of 16 bits or $\sim 64,000$, and cameras have as many as a few thousand columns. Thus MUD TADPOLE should be able to measure pulses with a TBP as large as $\sim 10^8$. For more complex pulses, clever methods for measuring oscillations of less than one count could improve its performance.

3. Data analysis

3.1 Spatial Fourier filtering

The first step in analyzing a MUD TADPOLE trace is spatial filtering [22,23]. It is essentially a Fourier band-pass filter that isolates the spatial-fringe signal term from the, in principle, spatially structureless background. In practice, the background can vary slowly along the spatial direction due to beam spatial mode structure, which can cause some of the background to "leak into" the retrieved signal. This excess background in the signal term is not a problem when the spatial-fringe signal is relatively strong, as is the case for simple pulses. But for complex pulses, the background term can become a significant contribution. In MUD TADPOLE, however, which uses single-mode fibers for the input pulses, the spatial mode is quite flat, and this effect is not a problem.

3.2 Temporal filter

In practice, we find it preferable to actually use delays smaller than the length of the spectrometer-broadened reference pulse in order to avoid using the weak leading and trailing regions of the product of the reference pulse and the retrieved temporal piece of the pulse. So, after the retrieved spectra for the various delays are Fourier transformed to the "time" domain, retrieved pulse information at both large and small delays is discarded as clearly not part of the piece of the unknown pulse.

3.3 Constant background subtraction

We also perform constant background subtraction before temporally filtering the data. We subtracted a constant background from the retrieved MUD TADPOLE spectrogram [Fig. 2(a) and Fig. 3(a)], and we found that this significantly reduces the high frequency noise in the retrieved temporal amplitude and phase. In the experiments described in this paper, we simply subtracted the maximum noise value from the retrieved spectrogram and then set any negative points that resulted from the subtraction to zero.

Although this background subtraction ultimately reduces the dynamic range, we still obtained excellent agreement between the retrieved and expected results.

4. Experimental setup

We performed experiments using a KM Labs Ti:Sapphire oscillator. The nearly-flat-phase ~28.5fs pulses (measured using a Swamp Optics GRENOUILLE, Model 8-20) were centered at 800 nm, with a FWHM bandwidth of ~40 nm. The pulses were stretched to a FWHM length of 70 ps using a grating pulse compressor.

The SEA TADPOLE set-up shown in Fig. 1 is described in more detail in [22]. Specifically, for our set-up we used a 100 mm focal-length spherical lens to collimate and cross the beams emanating from the fibers. Additionally, a 600groove/mm grating and 100mm focal-length lens were used for mapping wavelength to position in the spectrometer. The delay stage used was a Newport M-IMS600CC Linear Stage with a Newport ESP300 single-axis controller. The total scanning range of the delay stage was 120cm, which provided MUD TADPOLE's high spectral resolution.

We did not perform any experiments that were limited by the dynamic range of the camera due to the difficulty in generating such a complex pulse. We know of only a few simple methods to increase the TBP of a pulse. The temporal length of the pulse can be increased through the use of a fiber, pulse shaper/stretcher/compressor, or etalon. Or, the spectral bandwidth of the pulse can be increased by a nonlinear optical process like self-phase-modulation. In our experiment, the limited pulse energies, 5nJ per pulse with a repetition rate of 85 MHz, prevented us from significantly increasing the TBP by a nonlinear-optical process.

Additionally, generating a complex pulse with a fiber is impractical. For example, to stretch a 50nm pulse to 3ns, yielding a TBP of ~70,000, requires 170m of standard fused silica fiber. Using more dispersive fiber reduces the path length but increases the cost significantly. Furthermore, to stretch the pulse this much using a standard grating compressor would require 70m of path length. Both methods prove experimentally challenging for any optical setup and especially so for SI, which requires equivalent path lengths for both the reference and unknown pulses.

The final method for stretching a pulse uses an etalon or Michelson interferometer, which "stretches" the incident pulse by generating multiple replicas. This method results in a relatively simple temporal structure compared with the other methods, but the resulting spectrum is highly oscillatory and hence quite interesting. Measuring it is, to our knowledge, far beyond the ability of all available techniques.

The compressor used was a grating compressor with a path length of 2m which stretched the incident pulse up to ~70ps in length. The etalon consisted of two partially reflecting mirrors with a 90% reflecting coating. The high reflectivity of the two mirrors was chosen to minimize the relative intensity difference of the pulses in the pulse train.

Like all SI-based techniques, MUD TADPOLE measures the spectral-phase *difference* between the reference and unknown pulses, rather than the spectral phase of the unknown pulse. For the reference pulse, we used the pulse directly from the oscillator, so that the phase difference that we measured with MUD TADPOLE was the phase introduced by the pulse compressor and the Michelson interferometer in the first experiment and the phase due to the etalon and the pulse compressor in the second experiment. However, the phase distortions in

the pulse directly from our Ti:Sapphire oscillator were negligible compared to those of the pulse we generated to test MUD TADPOLE. So, in our analysis, we neglected any phase distortions in the reference pulse.

5. Results and discussion

We performed two experiments to demonstrate MUD TADPOLE's unique capabilities compared to other pulse characterization techniques. In both experiments MUD TADPOLE provides the necessary spectral resolution to completely characterize the intensity and phase difference between the unknown pulse and the reference pulse.

In the first experiment, we measured a double pulse consisting of two linearly chirped pulses stretched to 70ps FWHM. Over the entire 120cm scanning range, we collected 2800 SEA TADPOLE traces, each having a different reference-pulse delay using the set-up shown in Fig. 1.

The spectrometer used in this setup had half the spectral resolution and twice the spectral range of the previous MUD TADPOLE setup [17]. As a result the reference pulse stretches in time to $\tau_{sp} \sim 4ps$ (rather than 8ps) inside the spectrometer. The reference pulses were separated in time by $\tau_{ref} = 2.0ps$. Since $\tau_{ref} < \tau_{sp}$, there was overlap with neighboring reference pulses, which minimized discontinuities during the concatenation routine. The half width at 1/e of the weighting function [17] was chosen to be equal to the temporal separation of the reference pulses, $\tau_G = 2.0ps$.

A spectrogram is an intuitive representation of the individual SEA TADPOLE measurements at many delays and is easily computed from them, and Fig. 2(a) shows the retrieved MUD TADPOLE spectrogram. The slope of the lines in the spectrogram indicates that each pulse in the train is heavily chirped. A quick glance at the spectrogram shows that each line has the same slope indicating that each pulse has an identical chirp value. This is expected because, before the Michelson interferometer, which served to make the double pulse, the pulse was chirped by the pulse compressor. This is confirmed by Fig. 2(b) which shows the MUD TADPOLE retrieved temporal profile of the pulse, in which the temporal phase of each pulse is almost identical.

Figure 2(b) displays the retrieved temporal profile of the chirped double pulse. The ratio of the measured intensities of each pulse in the double pulse was 0.6. Using a power meter, the ratio of the intensities of the two pulses in the double pulse was found to be 0.8. This discrepancy is likely due to misalignment of the Michelson interferometer, yielding better coupling of one pulse than the other into the optical fiber.

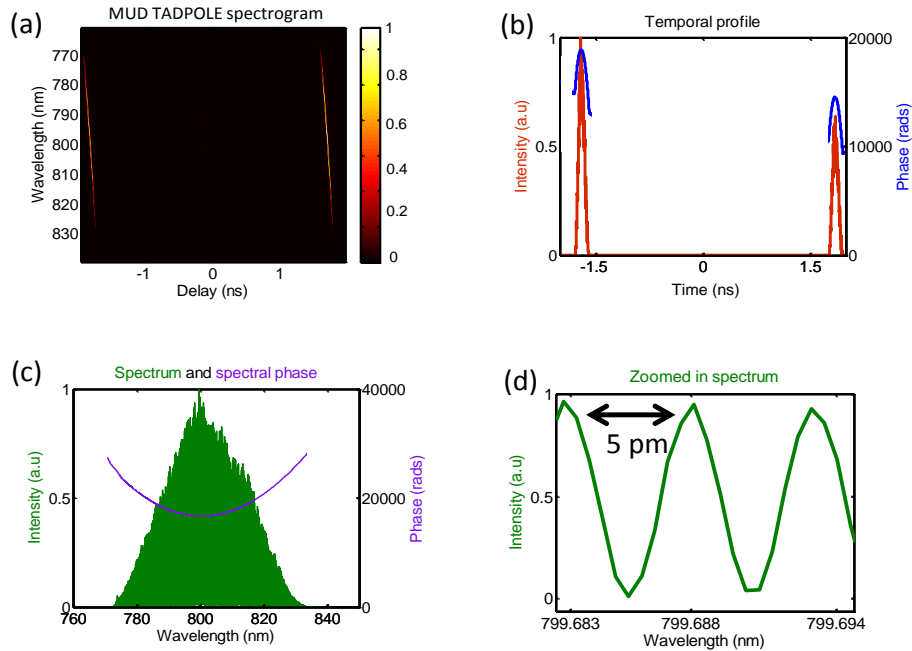


Fig. 2. a. The MUD TADPOLE spectrogram of a 3.5 ns chirped double pulse. b. The retrieved temporal intensity and phase of a 3.5 ns pulse. c. The MUD TADPOLE-retrieved spectrum. The solid color of the spectrum is due to the massive fine spectral structure in the complex pulse, which MUD TADPOLE is able to resolve. d. A zoomed in plot of a small section of the spectrum demonstrates MUD TADPOLE's high spectral resolution. The periodicity of the fringes in the spectrum was 5 pm.

Figure 2(c) highlights the high spectral resolution of MUD TADPOLE. The fringes are so fine that there is not sufficient spatial resolution on the page to reveal them all. Figure 2(d) shows an enlarged region of the spectrum, which illustrates that the fringe spacing is 5 pm. Such fine features in the spectrum have until now only been measurable with a very high resolution etalon spectrometer.

Our next experiment highlighted MUD TADPOLE's dynamic range and ability to measure even more complicated pulses. In this experiment, we measured a train of chirped pulses.

The train of pulses was generated by placing a mirror pair, each with a 90% partially reflecting face, after the grating pulse compressor. The mirrors were not precisely parallel, but still yielded a train of pulses at their output. As in the previous experiment, each pulse in the train had a FWHM temporal width of 70ps and a FWHM spectral bandwidth of 40nm.

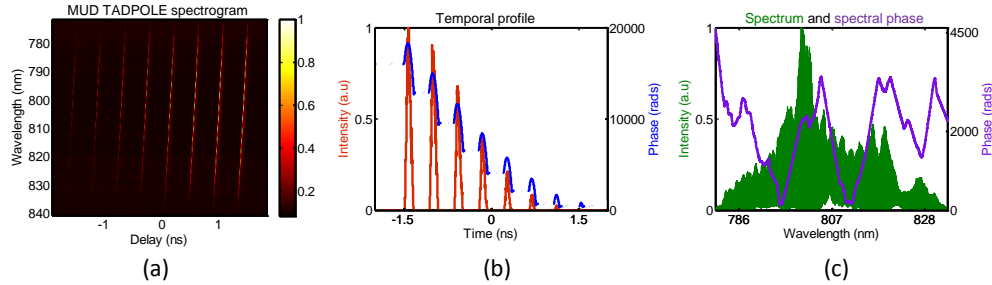


Fig. 3. a. The MUD TADPOLE spectrogram of a train of linearly chirped pulses. b. The temporal profile of the train of pulses. The measurement shows the steadily decreasing intensities of the pulses, the expected result of the multiple reflections inside the etalon. c. The spectrum of the pulse train. As expected, the asymmetric spectrum results from the nonparallel mirror pair and the differing absolute phases of the individual pulses in the pulse train. A more detailed view of the complex spectrum is shown in [Media 1](#), which is a movie that scans the spectrum along the entire spectral domain.

Figure 3(a) shows the retrieved MUD TADPOLE spectrogram. As in the previous figure, the slope of the lines in the spectrogram indicates that each pulse in the train is heavily chirped. This is expected because, before the mirror pair that generated the pulse train, the pulse was chirped by the pulse compressor. This is confirmed by Fig. 3(b), which shows the MUD TADPOLE retrieved temporal profile of the pulse.

The measured intensities of the pulses in the pulse train decrease in time, as expected. Although the temporal profile of the measurement in Fig. 3(b) is relatively simple, the spectral profile is complex, and MUD TADPOLE should be capable of measuring pulses with even more complex temporal profiles because it has proven capable of measuring pulses with 622fs substructure [17].

Figure 3(c) shows the retrieved spectrum of the pulse train, which exhibits MUD TADPOLE's large spectral range: $\sim 50\text{nm}$ in this measurement. A striking feature of the spectrum is its complex shape. In contrast to the spectrum of a chirped double pulse [Fig. 2(c)], which has a Gaussian envelope, the spectrum shown in Fig. 3(c) is much more complex. The unique shape is due to two factors. First, the two partially reflecting mirrors were deliberately aligned not to be parallel, in order to avoid back reflections back into the laser. This slight misalignment results in a different temporal spacing between the adjacent pulses in the pulse train, which corresponds to different spectral-fringe periodicities in the spectral domain. This is in contrast to the measurement of the double pulse in which there is only one periodicity in the spectral fringes due to the single temporal spacing between the two pulses. Second, the relative phase of each individual pulse in the pulse train differed, which shifted the spectral fringes due to each pulse in the train of pulses, and which served to further distort the envelope of the spectrum.

An instructive way to view the complex spectral structure in Fig. 3(c) is by scanning the spectrum along the wavelength axis. Figure 3(c) ([Media 1](#)) is a movie that simultaneously highlights MUD TADPOLE's high spectral resolution and large spectral range. The movie shows the complex spectral structure that can result from a seemingly noncomplex pulse in time, Fig. 3(b).

The TBP for both MUD TADPOLE measurements (Fig. 2 and 3) was $\sim 65,000$. Our setup was actually limited by the range of delay, rather than dynamic range of the camera, and the same experimental setup could have measured a much more complicated pulse, had we generated it.

6. Conclusion

We have demonstrated the first general technique for the measurement of complex pulses with TBP exceeding 65,000, fs temporal resolution, ns temporal range, pm spectral resolution, and nm spectral range. This simple and inexpensive device could also be used to accurately characterize seed pulses used in CPA systems and arbitrary optical waveforms with TBPs as large as 10^8 . Indeed, the parameters of our device fairly closely match that required to measure an arbitrary optical waveform from a Ti:Sapphire oscillator with a pulse separation of several ns and should allow measurements of extremely complex waveforms in general.

Acknowledgements

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