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Directly Recording Diffraction Phenomena in Time Domain[1]

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Abstract—By making use of a new technique for measuring the complete spatiotemporal electric field of light with micrometer spatial and femtosecond temporal resolution, we directly demonstrate the formation of the so-called boundary diffraction wave and Arago's spot, as well as the superluminal propagation of a "diffraction-free" pulse. We believe that such spatiotemporally resolved measurements and the time-domain treatment of diffracting waves not only turn out to be useful for modern physical optics, especially in micro- and meso-optics, but also significantly aid in the understanding of diffraction phenomena in general.

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1. INTRODUCTION

The bending of light waves in the shadow region behind an opaque disk and the appearance of a bright "Spot of Arago" in the shadow's centre are manifestations of diffraction and have been known for centuries. While the theory of diffraction, especially for monochromatic waves, is rather well developed, the subject has recently encountered some intriguing issues and new directions. The discovery of superluminally propagating "diffraction-free" wave-packets; the renaissance of an almost forgotten alternative interpretation of diffraction by the notion of the "boundary wave;" and the recent possibility of ultrashort-pulsed illumination all require some revisiting of the classical subject of optics.

Since the first observation of the diffraction of light in the middle of the 17th century by R.M. Grimaldi, tremendous progress was made in the mathematical treatment of the phenomenon, resulting in the well developed theory with Fresnel–Kirchhoff and Rayleigh–Sommerfeld versions (see, e.g., monographs [1, 2] and references therein). However, the approximations made in these theories, especially the neglect of the perturbations of light waves near the boundaries of openings and the treatment of obstacles as infinitely thin and made from totally absorbing or infinitely conducting idealized materials, remain under discussion. In particular, such approximations fail in near-field and subwavelength-geometry conditions, where material surface excitations play a decisive role (see, e.g., review [3]).

In the beginning of the 18th century, Thomas Young proposed, somewhat intuitively, that the diffraction pattern arises from the interference between the incident light propagating rectilinearly in accordance with the laws of geometrical optics and an omni-directionally propagating secondary wave originated from the edge of the diffracting body. On the other hand, Fresnel's theory—relying on Huygens' quite counter-intuitive assumption that each point of the incident wave front is a fictitious source of the secondary wave—proved more successful and consigned to oblivion Young's idea. It was rediscovered in 1888 by Maggi [4] and only in the middle of the last century was Young's idea developed into the theory of the boundary diffraction wave (BDW) by Rubinowicz [5], Miyamoto and Wolf (references given in [2]). This theory helped to resolve several issues of the standard Fresnel–Kirchhoff theory, to which it is mathematically equivalent, at least in the case of plane or spherical incident waves. In addition, calculation of diffracted fields according to the BDW theory is much less cumbersome than in the standard theory because only a simple contour integration along the opening boundary need be performed instead of a two-dimensional integration over the whole area of the opening, where Huygens' fictitious sources are located. Results of Sommerfeld's seminal rigorous calculation of diffraction of the electromagnetic field by a straight-edge—one of the few exactly solved diffraction problems—can easily be interpreted in terms of BDW. Yet, the BDW theory has remained outside the mainstream treatment of diffraction.

Nevertheless, diffraction from openings in opaque screens is well described and understood by the notion of the BDW theory. Especially intuitive should be the formation of a diffracted field in the case of illumination by ultrashort laser pulses, available in recent decades (see Fig. 1). Contrary to the traditional treatment using monochromatic fields, in which the transmitted waves fill large depths of space behind the screen and overlap with each other there, ultrashort pulses—typically femtoseconds long—are spatially only few micrometers "thick" and therefore behave almost like a solitary wave-front surface. Hence, the time-domain study of diffraction in terms of pulsed

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BDWs is not only didactically preferable but also opens new interesting directions and applications, such as in the study of focusing and other transformations of ultrashort pulses (see, e.g., paper [6] and references therein). The formation of an ultrashort boundary wave pulse on a circular aperture has been theoretically studied [7] and evidence for its existence experimentally identified by measuring modulations in the spectrum of the on-axis field and in CCD-recordings of the time-integrated radial intensity distribution of the field [8]. Our aim has been to directly record, with simultaneous spatial and temporal resolution, the evolution and interference of the boundary waves behind various screens.

In 1987 "diffraction-free" light beams [9] were introduced and now constitute a mature field with numerous applications (see review [10]). These beams—also known as Bessel beams—possess a controversial quality: they preserve their tightly focused central bright spot over large distances of propagation as if the beam does not obey the laws of diffraction.

At the same time, quite independently, the topic of undistorted or localized waves emerged in mathematical physics and deals with ultrabroadband pulses that are not only "diffraction-free" in space but also propagate without any spread in time [11–13]: "light bullets" and "electromagnetic missiles." To date, various localized waves propagating in vacuum superluminally, luminally (i.e., with velocity $c = 299792.458$ km/s), or subluminally have been studied in detail, and promising applications have been proposed (see, e.g, reviews [14–17] and the first monograph [18] on the field). The feasibility of such light bullets moving faster than c has been experimentally demonstrated more than once [19–24], but, from time to time, papers still appear in which the superluminal group velocity of such wave-packets in vacuum is questioned. Therefore, our second aim has been to accomplish, for the first time, with appropriately high resolution and accuracy, a direct spatiotemporal measurement of the electric field and propagation velocity of the simplest superluminal localized wave—the so-called Bessel-X pulse [19], which comprises an energy lump of micrometer diameter at the joint apex of a sparse double-conical wave.

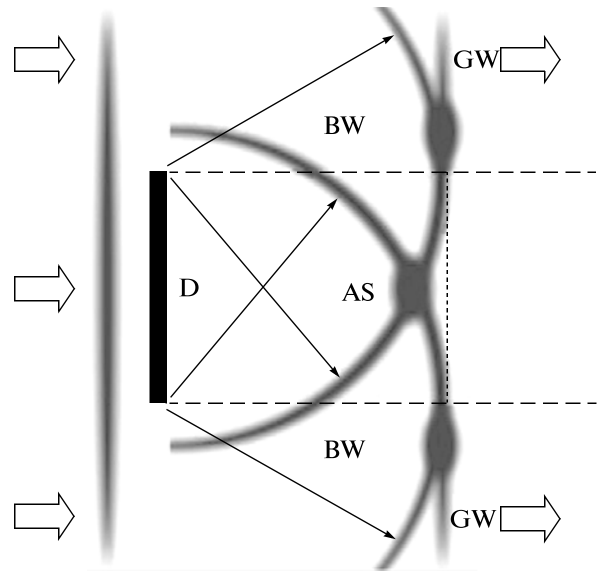

Fig. 1. Schematic of the formation of the Arago spot in the case of illumination with ultrashort pulses. A pancake-shaped pulsed wave hits a disk-shaped obstacle (D) from the left. On passage of the wave, the obstacle cuts off its central part according to the shadow boundaries (horizontal dashed lines) forming the geometrical wave (GW) component of the output field. Each point on the edge of the obstacle emits a secondary spherical pulsed wave as indicated by arrows, together forming the boundary diffraction wave (BW), which expands from a ring torus shape trough a spindle-torus-like stage (cross-section depicted in the figure) into a spherical wave at infinity. On the axis, overlapping and interfering boundary waves form the Arago spot. Around the shadow boundary in the overlap regions (also indicated by red ovals) of the BW and GW the common interference rings appear. The Arago spot (AS) propagates behind the front (indicated by vertical dashed line) of the transmitted GW but catches up with the latter at infinity because its velocity is superluminal.

2. SPATIOTEMPORAL MEASUREMENT OF LIGHT FIELDS

Our measurements not only required high spatiotemporal resolution, but also high sensitivity. First of all, we routinely measure the relatively high-intensity spatially uniform reference pulse, which is the pulse directly out of our laser, using a technique called FROG (Frequency Resolved Optical Gating [25])—which utilizes nonlinear optics and a sophisticated inverse algorithm to retrieve the pulse's field in time. To obtain ultrahigh temporal resolution in conjunction with the required sensitivity, we used a technique called SEA TADPOLE (Spatially Encoded Arrangement for Temporal Analysis by Dispersing a Pair of Light E-fields [26]), which is based on spectral interferometry. It involves measurement of the spectrum of the sum of the known reference pulse and the unknown pulse to yield the unknown pulse's temporal field, much like monochromatic-beam spatial interferometry or holography, where measurement of the spatial intensity of the sum of a known spatial field and an unknown monochromatic wave yields the unknown wave field in space. A detailed description of SEA TADPOLE can be found in [27–29]. Finally, we achieved high spatial resolution by simply scanning the micrometer-sized tip of the SEA TADPOLE input fibre point-by-point through the space where the unknown light field propagated.

3. SPATIOTEMPORALLY RECORDED DIFFRACTION

Here we show two of our results on diffraction of pulses through screens. The plots in Figs. 2, 3, 5 can be

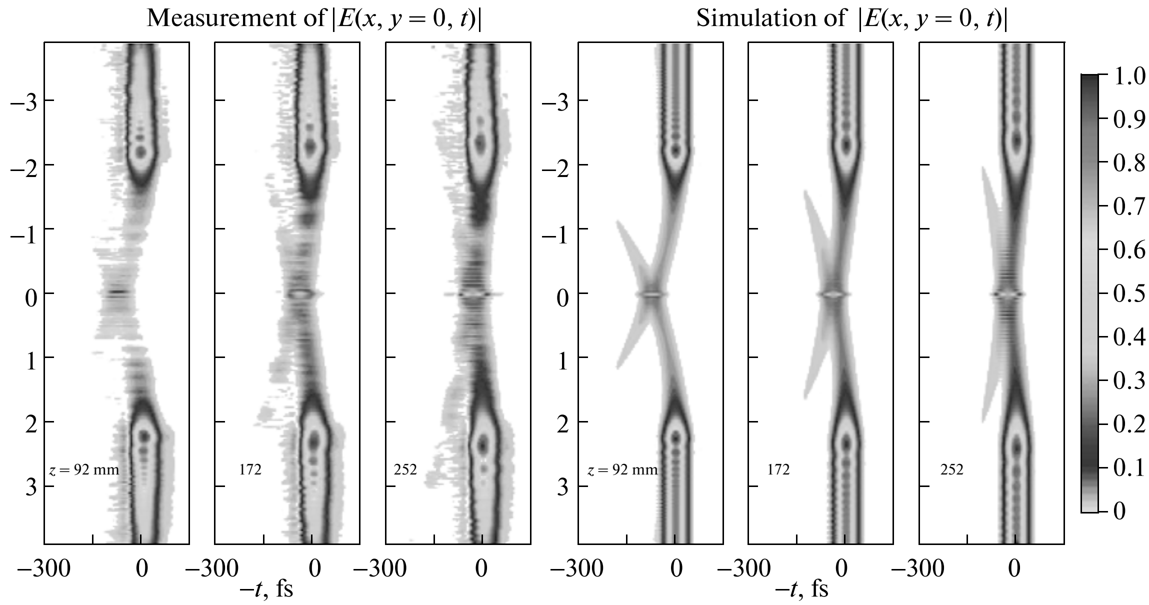

Fig. 2. Formation and evolution of the Arago spot behind an opaque disk of 4 mm in diameter. The magnitude of the electric field E is shown at three different propagation distances z in pseudo-color code according to the color bar (white has been taken for the zero of the scale in order to better reveal areas of weak field). In the measured plots the amplitude of the GW decreases with the transversal coordinate as the input pulses had a Gaussian radial profile, and in the simulations flat plane wave pulses were assumed according to our aim—to demonstrate how the simplest formula of the BDW theory describes properly the diffracted wave.

viewed as still images or "snapshots in flight," since they are spatiotemporal slices of the magnitude of the electric field $|E(x, y, z, t)|$ behind the screen. While we measured the complete pulse electric fields (amplitude and phase), we show only plots of the pulse amplitude because the phase of these pulses is less interesting. We show pulses measured at different propagation distances z. For comparison, theoretical simulations are presented on the right-hand side, which are carried out using the one-dimensional integral formula [7] of the boundary diffraction wave theory (the two-dimensional formula of the common diffraction theory gave the same results as expected).

First, according to the schematic in Fig. 1, we propagated ultrashort pulses past an opaque disk of 4 mm diameter, making a hole in the beam, and we measured the resulting spatiotemporal field at different distances from the aperture to observe its evolution. These measurements reveal the spatiotemporal structure of the weak boundary waves and the brighter spot at the centre of the beam due to their constructive interference, i.e., the spot of Arago, as it is known in conventional diffraction theory. Interestingly, the plots in Fig. 2 reveal that this spot is surrounded by coaxial interference rings and, in the axial region the field, generally resembles the Bessel-X pulse (considered in the next Section). Moreover, as shown in Fig. 2, the spot is indeed delayed in time with respect to the main pulse front, and this delay decreases with z, indicating a superluminal propagation speed along the z axis (the main pulse front propagates at c), which has been observed indirectly in a previous study [8] (where a spherical initial wave was used). This occurs, because, as z (or the distance from the disk) increases, the extra distance that the boundary waves must propagate (compared to the main pulse front) to reach the z axis ($x = 0$) decreases, so the relative delay of the boundary waves and the bright spot due to their interference decreases. In fact, the group velocity of the Arago spot—geometrically located at one pole of a luminally expanding spindle torus—varies from infinity at $z = 0$ to c for very large values of z. Thus it is an example of a decelerating light pulse.

Next, using the same initial field parameters, we propagated the beam through a 4.4 mm diameter steel circular aperture and measured the resulting diffraction. Again our measurements (Fig. 3) are in good agreement with the simulations, but with a minor discrepancy in the brightness of the main pulse front, which is likely due to the thickness (3.1 mm) and imperfect surface quality of our aperture. These measurements show a boundary-wave pulse behind the main pulse-front in time that eventually catches up with it. The boundary-wave pulse in these measurements looks very similar to that shown in Fig. 2. (The boundary waves in these two measurements look a little different because all of the images are normalized to have maximum of 1, and the main pulse front is much brighter in Fig. 3.) In fact, according to the boundary wave theory of diffraction, because the aperture and disk have similar diameters, their boundary waves are almost the same (but of opposite sign of the wave-function in accordance with the Babinet principle). So, interestingly, the Arago spot occurs due to

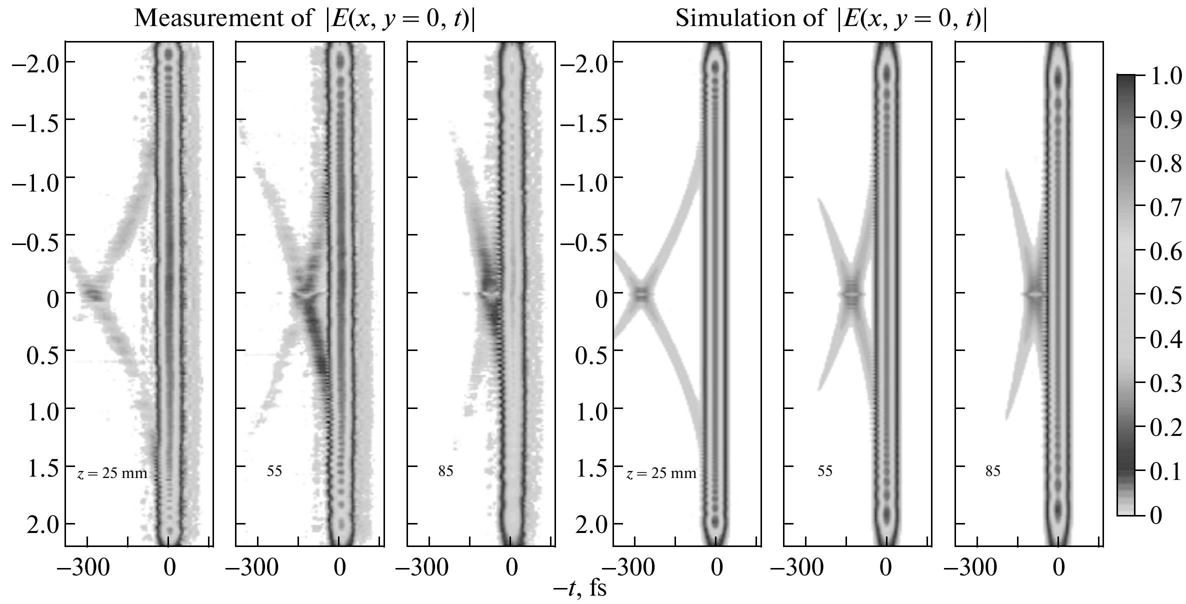

Fig. 3. Formation and evolution of the diffracted field behind a circular hole 4 mm in diameter. The boundary waves interfere with each other and with the directly transmitted pulse, but the interference maximum on the axis (actually a temporally resolved spot of Arago) lags behind the direct pulse, and eventually catches up with it.

any circular boundary and not just a circular disk. Moreover, due to the temporal localization of the pulsed illumination and the temporal resolution of our measurements, we can directly visualize the small delayed spot, which with longer pulses or continuous radiation would have overlapped with the intense, undiffracted beam.

4. RESULTS ON "DIFFRACTION-FREE" BESSEL-X PULSE

The most effective Bessel beam generator—a conical lens (axicon)—refracts plane waves towards the axis and thus shapes a femtosecond pulse into the Bessel-X pulse with its characteristic double-conical profile, as shown in Fig. 4. If the aperture radius R of the axicon were infinitely large, the pulse would propagate rigidly and without any spread of its micrometer-size central bright spot at the joint apex of the cones over an infinitely large distance. In the case of a limited aperture, it follows from the geometry in Fig. 4 that the depth of the invariant propagation of the pulse (let us call it the Bessel zone) is restricted to $z_B = R/\tan\theta$, where θ (the so-called Axicon angle) is the angle of inclination of rays toward the axis z.

Some such measured "snapshots" are shown in Fig. 5, together with theoretical simulations (this time calculated as axisymmetric superposition of plane waves with Gaussian aperture). The two are in good agreement except that the wings in the $z = 5.5$ cm image are shorter in the measurement. This is because axicons are difficult to machine perfectly; in particular, the tip of the cones are always distorted and generally the Bessel zone is shorter than what would be achieved in the ideal case.

There are several interesting features in these plots. The central maximum of the pulse has a width of ~20 μm, which—as well as the coaxial intensity rings surrounding it—remains essentially unchanged in shape from $z = 5$ cm through $z = 13.5$ cm. Thus the apex flies rigidly as a light bullet together with its sparse wings at constant speed. This is because the Bessel-X pulse is a propagation-invariant conical wave in distinction to the expanding toroidal wave forming the

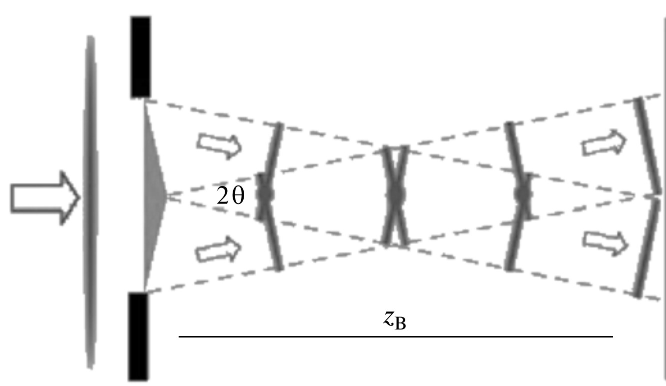

Fig. 4. Schematic of the formation of the Bessel-X pulse as a conical wave in the case of illumination of a conical lens (axicon) with ultrashort pulses. z_B indicates the range along the propagation axis, where the pulse can be considered as "diffraction-free." Ovals indicate the apex regions where interference and formation of the Bessel ring pattern take place.

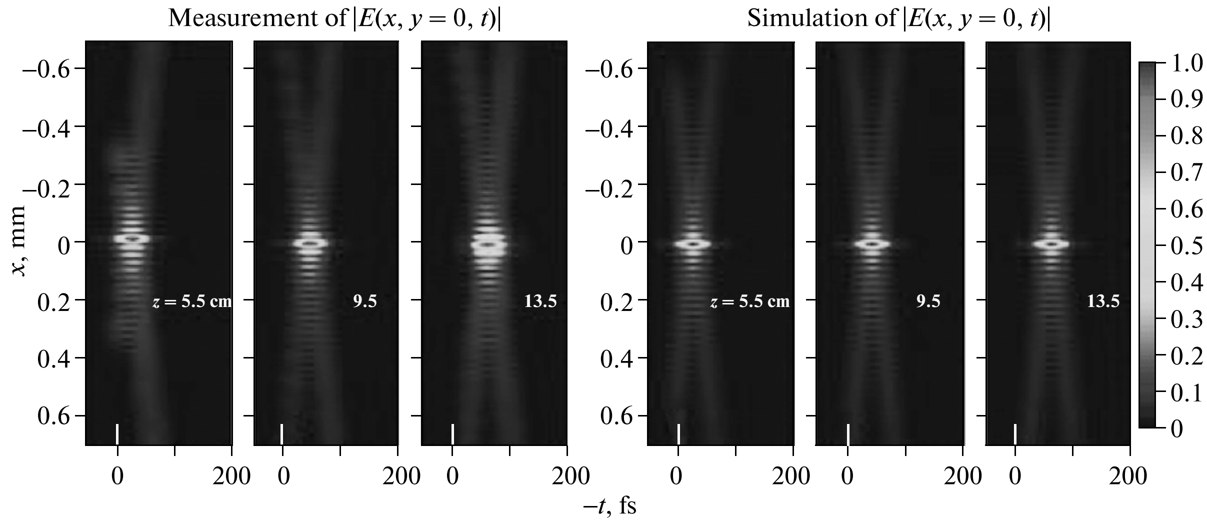

Fig. 5. Propagation of the Bessel-X pulse at three different distances after the axicon. The field magnitude in each plot has been normalized to have a maximum of 1. The white bar is to emphasize the location of a $t = 0$ of the luminally co-propagating reference.

Arago spot, which possesses approximately the Bessel profile (with expanding rings) near the axis only.

Also, the Bessel X-pulse's superluminal speed is apparent in these plots. SEA TADPOLE measures the pulse's arrival time with respect to the reference pulse, which travels at the speed of light (c). Therefore, if the Bessel X-pulse were traveling at the speed of light, then at each z its spatiotemporal intensity would be centered at the same time (here $t = 0$ and emphasized with the white line), but it is easy to see that this is not the case. From our axicon's angle θ (and from the simulations), we find that the Bessel-X pulse's speed (axial group velocity) should be $1.00013c$. From our experimental plots we determined [29] it to be around $1.00012c$—within 0.001% error of the expected value.

5. DISCUSSION

The superluminality of the Bessel-X pulse and that of the Arago spot pulse are intriguing. Indeed, while phase velocities greater than c are well known in various fields of physics, a superluminal group velocity more often than not is considered as a taboo, because, at first glance, it seems to be at variance with relativistic causality. However, thanks to the numerous studies throughout the previous century—starting from Sommerfeld's works on propagation of plane wave pulses in dispersive media—it is well known (see, e.g., a thorough review [30]) that the group velocity need not be a physically profound quantity and by no means should be confused with the signal propagation velocity. But in the case of Bessel-X-type pulsed waves no dispersive medium need to be involved and still, not only is the group velocity superluminal, but the pulse as a whole rigidly propagates faster than a plane wave.

Naturally, one feels some unease in accepting this startling circumstance. But here we experimentally observe it in the most direct way. When forced to concede the theoretically and experimentally verified superluminality, one might feel the need to make recourse to statements insisting that the pulse is not a "real" one, but instead simply an interference pattern rebuilt at every point of its propagation axis from truly real plane wave constituents travelling at a slight tilt with respect to the axis. Such argumentation is not wrong but, alas, it leads nowhere. Of course, there is a similarity between the superluminality of the X-wave and a simple geometrical faster-than-light movement of the cutting point in scissors (we refer here to Gedanken experiments described in textbooks on relativity). But in the central highest-energy part of the Bessel-X wave, there is nothing moving at the tilt angle. The phase planes are perpendicular to the axis and move rigidly with the whole pulse along the axis. The Poynting vector, indicating the direction of energy flow, lies also along the axis. However, the energy flux is not superluminal. Hence, to consider the Bessel-X waves as something inferior to "real" waves is not sound. If we thought so, by similar logic we would arrive at the conclusion that femtosecond pulses emitted by a mode-locked laser are not real but "simply an interference" between the continuous-wave laser modes. In other words, one should not ignore the essence of the superposition principle of linear fields, which implies a reversible relation between "resultant" and "constituent" fields and does not make any of the possible orthogonal bases—plane waves and cylindrical (Bessel) waves, for the given example—inferior to others.

Another misunderstanding (the author of the review [30] seems to agree) stems from oversight of the fact that there are infinitely many ways to form a pulsed wave-packet from single-frequency Bessel beams. They depend on how the radial density of intensity rings in the beam cross section is related—or whether or not it is related at all—to the beam's temporal frequency. In the case of the Bessel-X pulse, this is a proportionality relation, and therefore the group velocity is perfectly defined with a single superluminal value within the whole bandwidth of the wave-packet. If, on the contrary, the radial density is frequency-independent, we obtain a completely different wave-packet, which does not belong to localized waves since it has no definite group velocity over its whole spectrum and therefore spreads in the course of propagation. But such a wave-packet—named the "pulsed Bessel beam" in the literature—propagates with velocity less than c and can be used for sending signals along the propagation axis. On the other hand, if one tried to cut a signal "notch" into the core of the Bessel-X pulse, the notch would behave like the "pulsed Bessel beam"—spreading out while advancing subluminally. This is natural, since Maxwell's equations or the wave equation for EM fields does not allow superluminal signalling.

6. CONCLUSIONS

By direct spatiotemporally resolved measurements of pulsed light fields behind the simplest diffracting screens we have shown for the first time experimentally how the transmitted wave-field is gradually formed as a superposition of the directly transmitted pulse and an expanding boundary wave according to the almost forgotten boundary diffraction wave theory. In particular, we observed the formation and superluminal, but decelerating, movement of a small peculiarity caused by interfering boundary waves, which is responsible for the appearance of the Arago spot in a common steady-state diffraction pattern. With appropriate and higher-than-in-previous-studies resolutions and accuracy we recorded directly the bullet-like propagation of a "diffraction-free" Bessel-X pulse and measured its superluminal speed. In summary, we believe that time-resolved measurements and time-domain treatment of diffracting waves not only turn out to be fruitful in modern physical optics, especially in micro- and meso-optics, but also promote the understanding of diffraction phenomena.

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